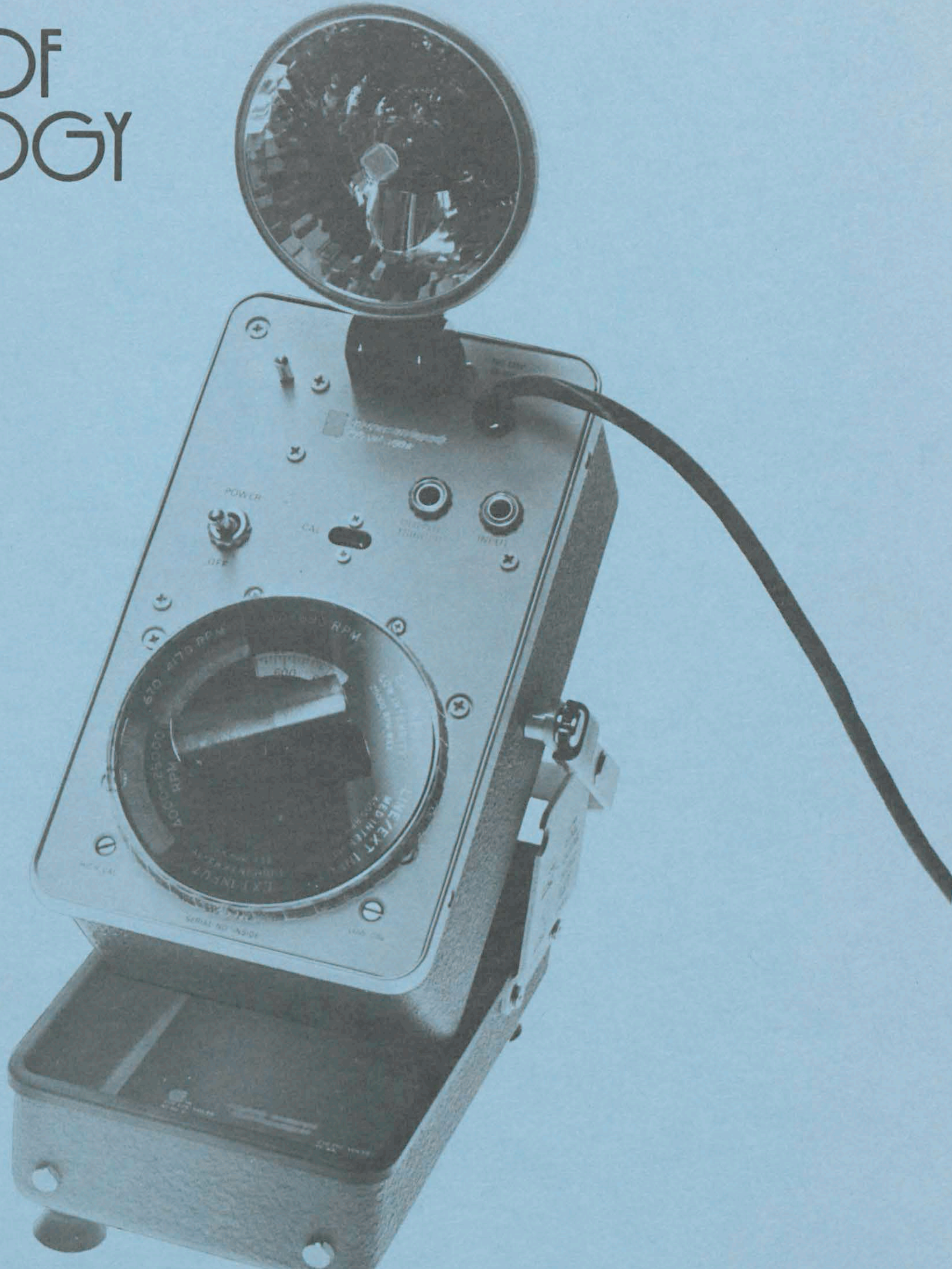
pt SUNY

PHYSICS OF TECHNOLOGY

COORDINATED BY AMERICAN INSTITUTE OF PHYSICS



THE STROBOSCOPE

Forces and Motion

THE STROBOSCOPE

A Module on Forces and Motion

SUNY at Binghamton

Carl R. Stannard, State University of New York at Binghamton
John P. Ouderkirk, State University of New York Agricultural and Technical College at Canton
Bruce B. Marsh, State University of New York at Albany

Carl R. Stannard, State University of New York at Binghamton
Bruce B. Marsh, State University of New York at Albany
Project Co-Directors

MCGRAW-HILL BOOK COMPANY

NEW YORK
ST. LOUIS
DALLAS
SAN FRANCISCO
AUCKLAND
DUSSELDORF
JOHANNESBURG
KUALA LUMPUR
LONDON
MEXICO
MONTREAL
NEW DELHI
PANAMA
PARIS
SÃO PAULO
SINGAPORE
SYDNEY
TOKYO
TORONTO

The Physics of Technology modules were produced by the Tech Physics Project, which was funded by grants from the National Science Foundation. The work was coordinated by the American Institute of Physics. In the early planning stages, the Tech Physics Project received a small grant for exploratory work from the Exxon Educational Foundation.

The modules were coordinated, edited, and printed copy produced by the staff at Indiana State University at Terre Haute. The staff involved in the project included:

Philip DiLavore . . . . Editor
Julius Sigler . . . . Rewrite Editor
Mary Lu McFall . . . . Copy and Layout Editor
B. W. Barricklow . . . . Illustrator
Stacy Garrett . . . . Compositor
Elsie Green . . . . Compositor
Donald Emmons . . . . Technical Proofreader

In the early days of the Tech Physics Project A. A. Strassenburg, then Director of the AIP Office of Education, coordinated the module quality-control and advisory functions of the National Steering Committee. In 1972 Philip DiLavore became Project Coordinator and also assumed the responsibilities of editing and producing the final page copy for the modules.

This module was written and tested at the State University of New York at Binghamton. In addition to the Project Co-Directors, the professional staff includes Arnold Benton, the American Institute of Physics; Malcolm Goldberg, Westchester Community College; Gabriel Kousourou, Queensborough Community College; Ludwig P. Lange, Broome Community College; John F. Ouderkirk, SUNY ATC, Canton, and Arnold A. Strassenburg, SUNY, Stony Brook.

The authors wish to express their appreciation for the help of many people in bringing this module to final form. The criticisms of various reviewers and the cooperation of field-test teachers have been most helpful. Several members of the staff of the State University of New York at Binghamton also deserve special recognition for their contributions. They are: Felix K. Chen, Duane A. Damiano, Eugene C. Deci, Barry R. DeGraff, Max Eibert, Steve I. Finette, Chung-Yu Fu, Jeffrey W. Gray, Leonard R. Kelly, Stevan H. Leiden, Debra M. Simes, Laurie A. Sperling, William Standish, and Howard M. Zendle.

The Stroboscope

ISBN 0-07-001722-0

1 2 3 4 5 6 7 8 9 0 E B E B 7 8 3 2 1 0 9 8 7 6 5

TABLE OF CONTENTS

The Stroboscope

INTRODUCTION

A *stroboscope* is a light source that can be turned on and off very quickly. The light can be flashed many thousands of times per minute or as slowly as a few times each minute. Each flash lasts about a millionth of a second.

You may have seen a stroboscope in operation. When an automobile is tuned up, a special stroboscope called a *timing light* is used to make sure that the spark arrives at the right cylinder at the right time. Proper timing of the engine is important to ensure that the gasoline is used efficiently, and with minimum pollution.

The stroboscope makes it possible for repeating motion to appear stopped. This happens if the light always flashes when the moving device is in the same position. If the position of the device when the light flashes is only slightly different from its position at the time of the previous flash (even though it may have made several rotations or cycles between flashes), the device appears to be moving more slowly than it actually is. You have seen this effect in the movies. The motion picture camera takes 24 separate pictures a second, so the movies you see at the theater have a built-in stroboscopic effect. When a wagon wheel speeds up or slows down, it seems to turn one way, stop, then turn the other way. This occurs because the camera only "samples" the motion of the wheel. What you see on the movie screen depends on where the wheel is when each picture is taken, but not on what it does between pictures.

Stroboscopes are used in live entertainment also. For example, they are used in discotheques to give special lighting effects. If you find yourself getting dizzy or sick in such a place, the flashing light may be the reason.

One of the most interesting and practical uses of the stroboscope is related to photography. High-speed events can be seen in detail if we take pictures using stroboscope flashes (see Figure 1). The use of a high-speed motion picture camera along with a stroboscope results in an extremely slow-motion movie of an event.

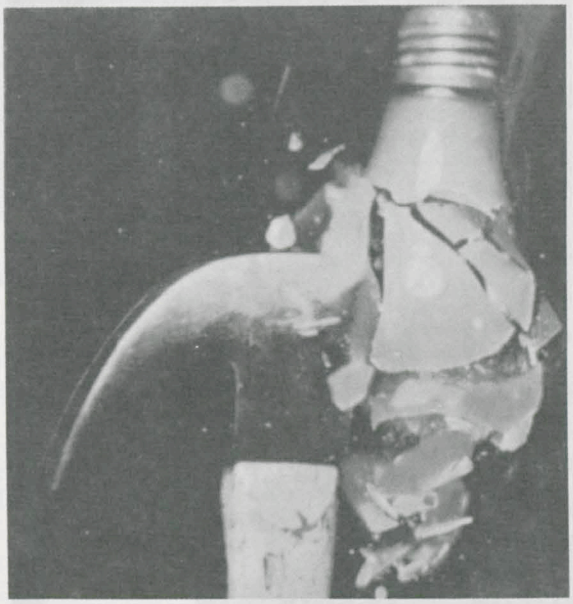


Figure 1. The very short duration of the stroboscope flash has resulted in some incredible photographs, like this one of a hammer breaking a light bulb. (Photo courtesy of General Radio Company.)

In physics the stroboscope/camera combination makes the study of motion easier. For example, a stroboscopic photograph of a falling object, such as Figure 2, offers a clear record of the event from which details of the motion can be determined.

To be useful in science and technology the flash rate of a stroboscope should be variable and measurable. Electronic stroboscopes, like that shown in Figure 3, satisfy both requirements.

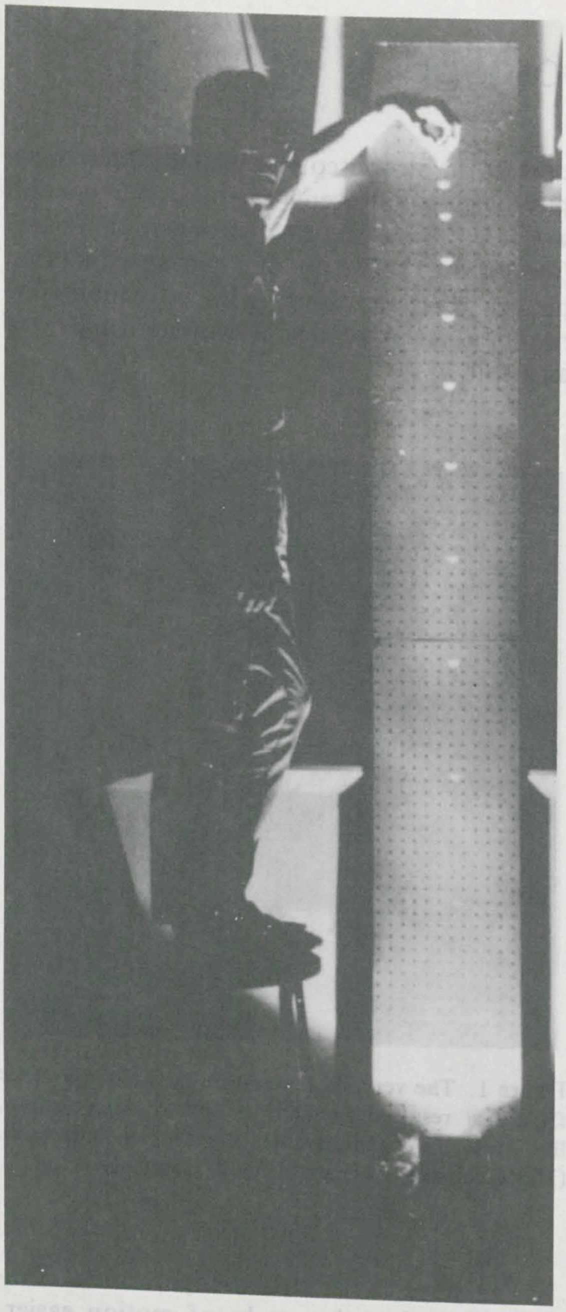


Figure 2. A strobe photo allows easy analysis of the motion of a falling ball.

GOALS OF THIS MODULE

Students often wonder why a particular topic is studied, and what is supposed to be learned. Experiences of both students and teachers have shown that when the goals of a course are made clear, they are more easily achieved by both the student and the teacher. The following questions are the most important points which we will try to answer in this module.

1. How does the stroboscope "freeze" motion, and how can one use this "frozen" motion to measure speeds of rotating objects or other repetitive motion?

2. If pictures are taken of moving objects using stroboscopic illumination, how can they be used to make graphs which show the important features of the motion?

3. How can one identify and measure some important characteristics of motion such as distance traveled, time during which motion occurs, speed, and the rate at which speed changes?

4. How can one analyze graphs constructed from strobe pictures to understand motion along a straight line?

5. How are algebraic equations used to calculate values for quantities which are not easily measured?

6. What causes the speed of an object to change? What do we need to know to predict how fast the speed will change?

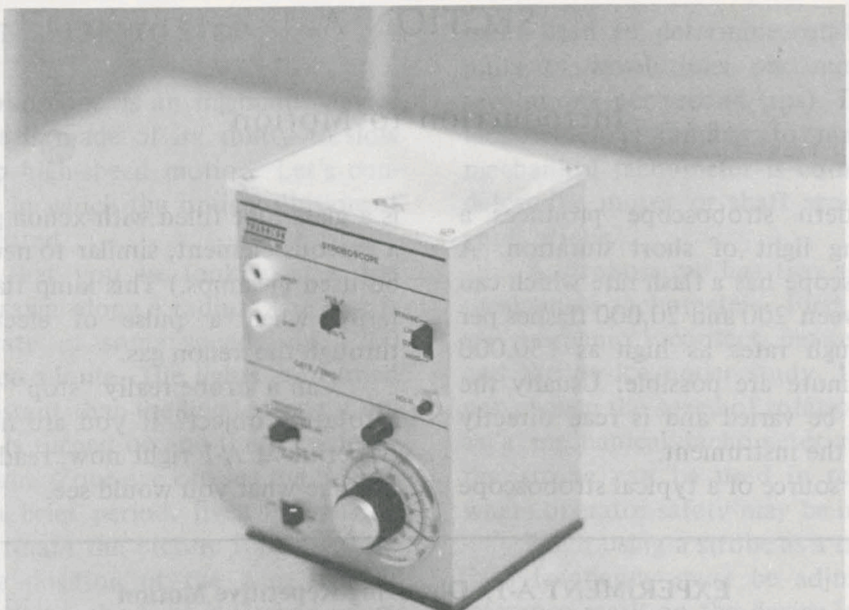


Figure 3. One type of stroboscope.

SECTION A

Introduction to Motion

The modern stroboscope produces a bright, flashing light of short duration. A typical stroboscope has a flash rate which can be varied between 200 and 20,000 flashes per minute, although rates as high as 150,000 flashes per minute are possible. Usually the flash rate can be varied and is read directly from a dial on the instrument.

The light source of a typical stroboscope is a glass tube filled with xenon gas. (Xenon is a gaseous element, similar to neon, which can be used in lamps.) This lamp flashes momentarily when a pulse of electricity passes through the xenon gas.

Can a strobe really "stop" the motion of a rotating object? If you are not able to do Experiment A-1 right now, read it and try to imagine what you would see.

EXPERIMENT A-1. Observing Repetitive Motion

This experiment illustrates basic techniques for measuring repetitive motion using the strobe.

CAUTION: Care should be taken to protect students and instructor from the hazards normally associated with power equipment. Safety glasses should be worn, and the devices used should be fastened down and shielded with clear plastic if possible.

Procedure

1. Make a mark or attach a piece of tape to provide a *reference mark* on the object to be observed.

2. Calibrate the strobe by following the directions supplied with it.

3. Shine the strobe on the moving object.

4. Vary the flash rate of the strobe and observe the changes in appearance of the object.

5. Starting with a low flash rate (a few hundred flashes per minute), increase the rate until you obtain a "frozen" pattern, with a single image of the reference mark. Can you find several other flash rates which produce the same "frozen" image? Find the *highest* rate at which a stationary single image is formed. Make a table for recording data and record this rate.

6. You should find another kind of stationary image at twice the highest rate found in step 5. Record the rate in your table and describe the image.

7. Double the rate again and find a stationary pattern. Record the rate and describe the image.

Question 1. Can you explain what is happening? Can you determine how fast the object is rotating?

8. Find several other flash rates at which stationary patterns with multiple images are formed. Record the rates and sketch the images.

9. Explain the results. If you have difficulty, the next section should help.

WHY MOTION SEEMS TO STOP

The stroboscope is an ingenious device and much use is made of its ability to slow down or stop high-speed motion. Let's consider the way in which the optical illusion of slowness is created.

Imagine that you are looking at a disc with a line drawn along a radius. The disc is made to rotate at some speed, say 1200 revolutions per minute. The lights are turned off. At the instant that the line is straight up, a bright light is turned on and then is quickly turned off again. Your eye can see the line on the disc for a brief period. Even though the eye does not retain the picture for long, you remember the position of the line. If after exactly one second, the light is flashed again, you see the line in the same position as before because the disc has made 20 complete revolutions. (Is that the correct number?) If the light flashes every half-second, the disc completes ten revolutions between flashes and the line is still always seen in the same place. The eye is fooled and sees the line as stationary. The disc is rotating, and the line rotates with it. However, it appears to stand still, since you always see the line in the same place.

THE STROBOSCOPE AS A TACHOMETER

In Experiment A-1 you observed that:

a. The stroboscope can create single or multiple images of a reference mark on a rotating object.

b. The flash rate of the stroboscope can be read from a calibrated dial.

c. The number of images depends on the relation between the flash rate and the speed of rotation.

These properties are the essential ingredients for a *tachometer.* A tachometer is an instrument used to determine rotational speed in units of revolutions per minute (rpm) or revolutions per second (rps). The automobile tachometer is familiar to many people. A mechanical tachometer is commonly used to determine motor or shaft speed in technical applications.

A stroboscope has two advantages over mechanical tachometers. First, since there is no mechanical contact between the strobe and the device under study, the strobe does not change the speed of rotation of the device as a mechanical tachometer might. Second, the strobe can be used in restricted places where operator safety may be important.

When using a strobe as a tachometer, the flash frequency must be adjusted so that a reference mark on the device is "held stationary" by the stroboscope light. Two rules are necessary to measure the speed.

RULE 1. If a reference mark on a rotating object is viewed with a stroboscope, a single stationary image will be seen if the rotating object makes a whole number (1, 2, 3, . . .) of complete revolutions between flashes. (See Figure 4A.)

Figure 4A. Single images.

Rule 1 is used when the flash rate is less than or equal to the rotation rate of the disc.

However, suppose the strobe flashes exactly twice for each rotation of the disc. This produces a different picture. Since the strobe flashes twice for each revolution of the disc, the line is lighted and seen at two positions which are 180° apart, as shown in Figure 4B.

If the strobe rate is increased to a flash exactly three times per revolution, then three images of the line are seen, and so on. Such situations are described by Rule 2.

RULE 2. When a reference mark on a rotating object is lighted by a stroboscope, a number of distinct, stationary images of the mark is seen if the flash rate of the stroboscope is a whole number (1, 2, 3, 4, . . .) multiple of the speed of rotation. (See Figure 4B.)

In order to use the stroboscope as a tachometer, one must be able to relate the number (n) of stationary images and the flash rate (f) to the rate of rotation (R) of the object being tested. Rules 1 and 2 state the conditions necessary to observe single and multiple images of a reference mark on a rotating object. For values of R greater than the flash rate f of the strobe, the strobe is adjusted for *single* stationary images (Rule 1); for values of R less than f, the strobe is adjusted for *multiple* stationary images (Rule 2). R can be computed by simple mathematical methods in either case. However, we will concentrate on the second case, since it is easier to apply and is more commonly used.

Whenever R is known to be less than or equal to the flash rate of the strobe

$$R = f/n$$

where n is the number of stationary images, f is the stroboscope reading in flashes per minute (fpm), and R is the rotational speed of the object in revolutions per minute (rpm).

The experimental procedure consists of adjusting the flash rate until stationary images are observed. The flash rate f and the number of images n are observed and recorded, as shown in Table I for our case of 1200 rpm. The calculated value of R should be the same for all cases.

It is possible, however, to obtain false readings. For example, consider an object rotating at 3000 rpm. A double image of the reference mark appears at a flash rate of 6000 fpm, and a triple image at 9000 fpm, etc. However, at a flash rate of 4500 fpm, the

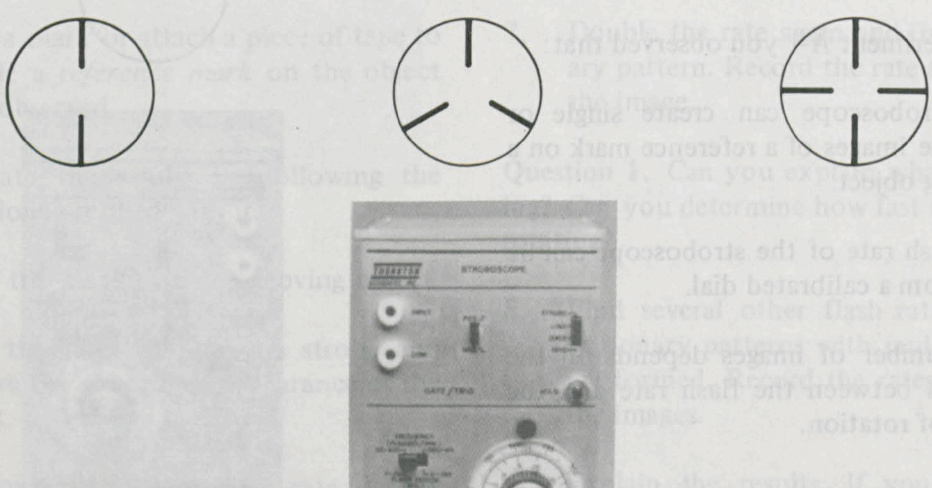


Figure 4B. Multiple images.

Table I.

f (fpm)	n	R (rpm)
1200	1	1200
2400	2	1200
3600	3	1200

reference mark makes two-thirds of a revolution between flashes. A triple image appears as shown in Figure 5.

Now the value of collecting the data in the form of a table should be clear. If the calculated values of R are the same for all the trials, the value is correct. If the values of R are different, what measurements could you make to learn the correct value?

At flash rates less than 600 fpm the eye has difficulty in retaining the image, and it appears to flicker. The ability of the eye to retain an image for a short time is called *persistence of vision.* Persistence of vision is useful to us when we use another stroboscope-like device, the motion picture projector. If the eye did not retain an image, the screen would appear to go blank between each successive frame of the movie. The flickering of images at low strobe speeds is the same phenomenon as the flicker seen in old movies which ran at slow speeds. The time between images is long enough that the images retained by the eyes start to fade before the next image appears.

Problem 1. A three-bladed floor fan has a piece of tape stuck to one blade as an identifying mark. The fan is illuminated with a stroboscope. A single stationary image of the tape is seen at 560 fpm and 1120 fpm. At 2240 fpm, two images 180° apart are seen. What is the speed of rotation of the fan?

Problem 2. A 3500-watt generator in a motor home recently was in the shop because the speed control (governor) was out of adjustment. A stroboscope was used. The flywheel was marked with chalk, and the following results were obtained: a single stationary image of the chalk mark was seen at 500, 750, and 1500 fpm. Three images of the chalk mark 120° apart were seen at 4500 fpm. The rotational speed should be 1800 rpm to deliver the rated power. Does the governor need to be adjusted? If so, how much (rpm) and in what direction?

Problem 3. A power handsaw is supposed to operate at 5500 rpm according to the nameplate. Students put a single reference mark on the blade and with a calibrated stroboscope obtained these data: single stationary images at 2700 fpm, 1800 fpm, and 1350 fpm; two stationary images, 180° apart, at 10,800 fpm. What was the actual rotational rate?

USES OF THE STROBOSCOPE

Both the speed-measuring capabilities of the stroboscope as a tachometer and its ability to slow down or "freeze" high-speed motion have been mentioned. Most applications of the stroboscope are related to these capabilities.

Let's consider first the ability of the stroboscope to create the optical illusion that high-speed motion has stopped or been "frozen." You saw this effect in Experiment A-1 where a strobe was used to measure rotational rates. In a similar fashion, one can

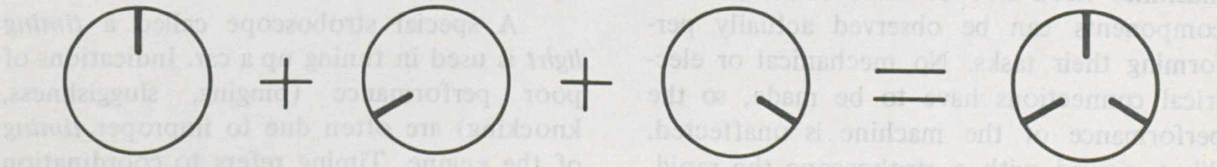


Figure 5.

observe moving machine parts to determine if those parts are operating properly. The printing and textile industries use stroboscopes to watch high-speed presses and weaving machines. If a single part in a complex, high-speed machine breaks, it may cause extensive damage and costly shut-down time. It pays to try to find defective parts before they cause trouble. Worn or poorly adjusted parts can be detected by stroboscopic inspection, and repairs can be made before breakdowns occur.

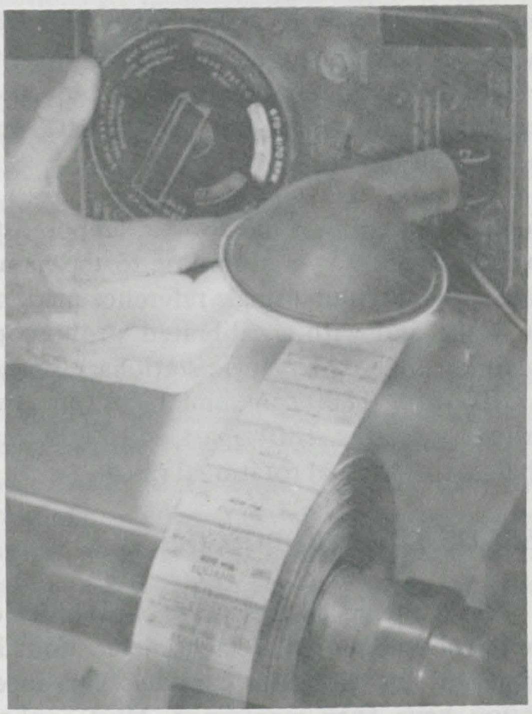


Figure 6. Photograph of a label-inspection process, with label strip moving at 1000 ft/min. (Courtesy of General Radio and Wyeth Laboratories, Inc.)

For example in Figure 7, the cam and follower mechanism must be operating for a malfunction to be observed. You cannot find the trouble by stopping the machine. The beauty of stroboscopic inspection is that machines need not be shut down and the components can be observed actually performing their tasks. No mechanical or electrical connections have to be made, so the performance of the machine is unaffected. When viewed with a stroboscope the rapid, complex movement of a machine seems to stand still or to take place in slow motion.

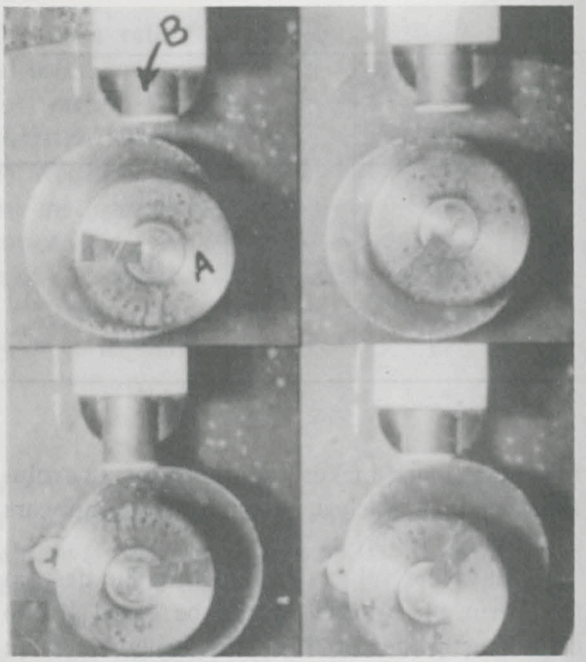


Figure 7. *Cam* (A) and *lazy follower* (B). The two parts are supposed to stay in contact and they do not in these photos. The cam is rotating at 3000 rpm. Note the piece of tape used as a reference mark. (Courtesy of General Radio Company.)

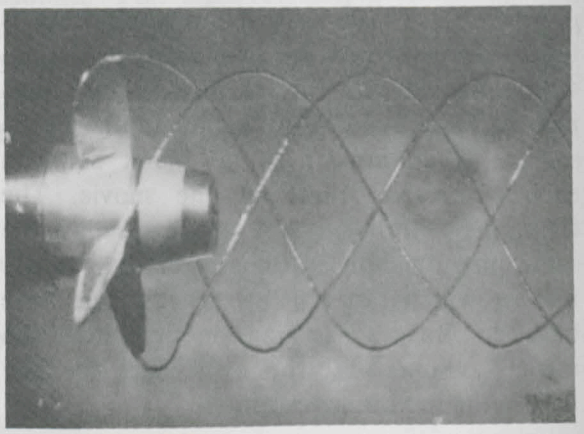


Figure 8. High-speed stroboscopic motion pictures show the turbulence patterns in the water as the propeller of a boat moves through it. (Courtesy of General Radio Company.)

A special stroboscope called a *timing light* is used in tuning up a car. Indications of poor performance (pinging, sluggishness, knocking) are often due to improper *timing* of the engine. Timing refers to coordination between the electric arc developed at the spark plug and the position of the piston in

the cylinder as it compresses the gas-air mixture. If the spark does not ignite the fuel at the right time, the engine does not perform properly and damage to some parts may result. A timing light is used to synchronize the spark with the correct piston position. The timing light shown in Figure 9 is just a flash tube in a gun-like housing. This timing light is a strobe light with *external synchronization.*

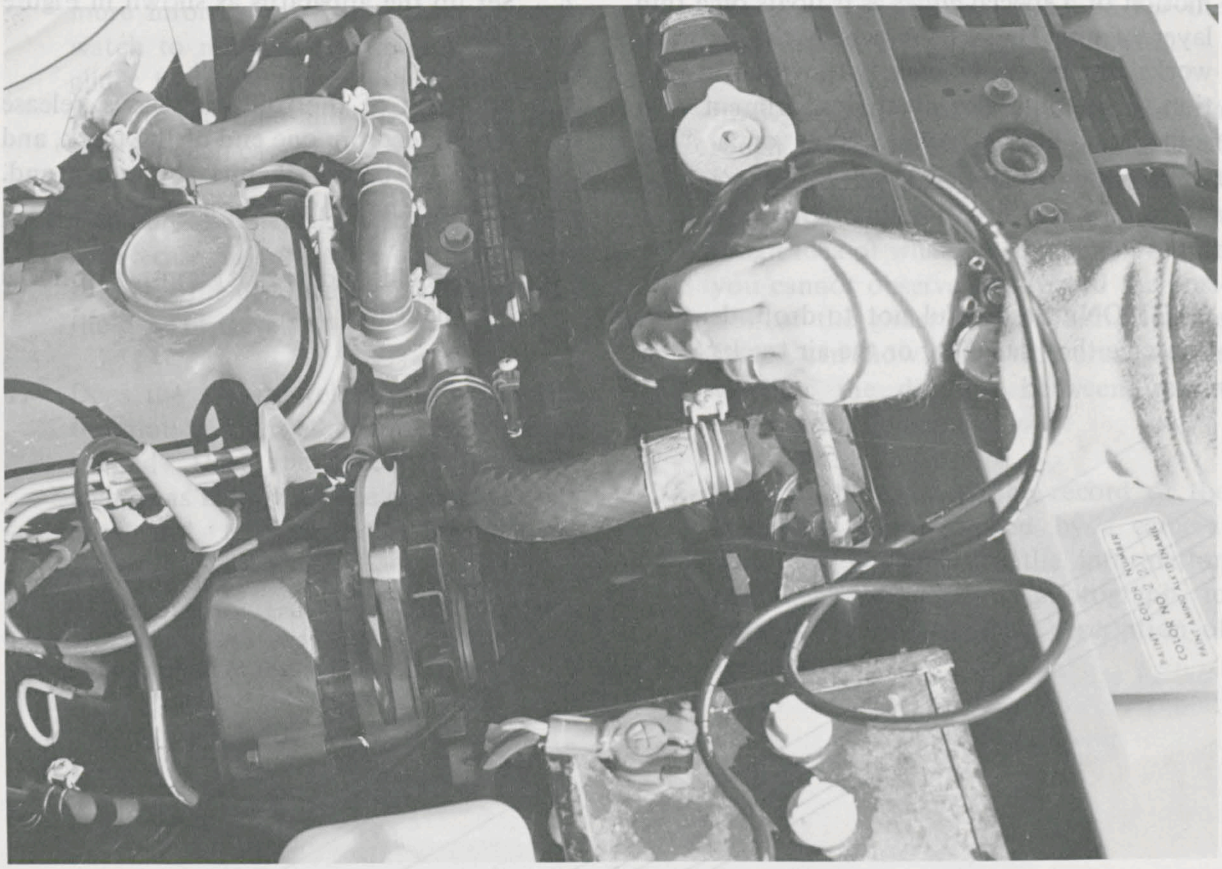


Figure 9. A timing light in use. The electrical energy is supplied either by the auto battery or from an AC outlet. The high-voltage pulse that makes one of the spark plugs fire (usually the number 1 plug) triggers the flash.

EXPERIMENT A-2. Observing Translational Motion

In Experiment A-1 you observed the motion of rotating objects. In this experiment you will observe the motion of objects moving along a line in *translational motion.* You will use an air track to study the translational motion of a special *glider* as it floats on a thin layer of air. The way in which the air track works can be easily seen by a visual inspection. You could also do the experiment with something like a roller skate or a "hot wheels" car.

Procedure

CAUTION: Be careful not to drop, dent, or scratch either the glider or the air track.

1. Turn on the air supply to the track. Set a glider on the track and give it a *gentle* push. Observe how the glider moves. Is the air track level? How do you know?

2. Set up the apparatus as shown in Figure 10.

3. For each of the following cases, release the glider from one end of the track, and catch it before it hits the other end. Observe the motion for each case.

   a. Small glider, 10 grams (g) on the hanger

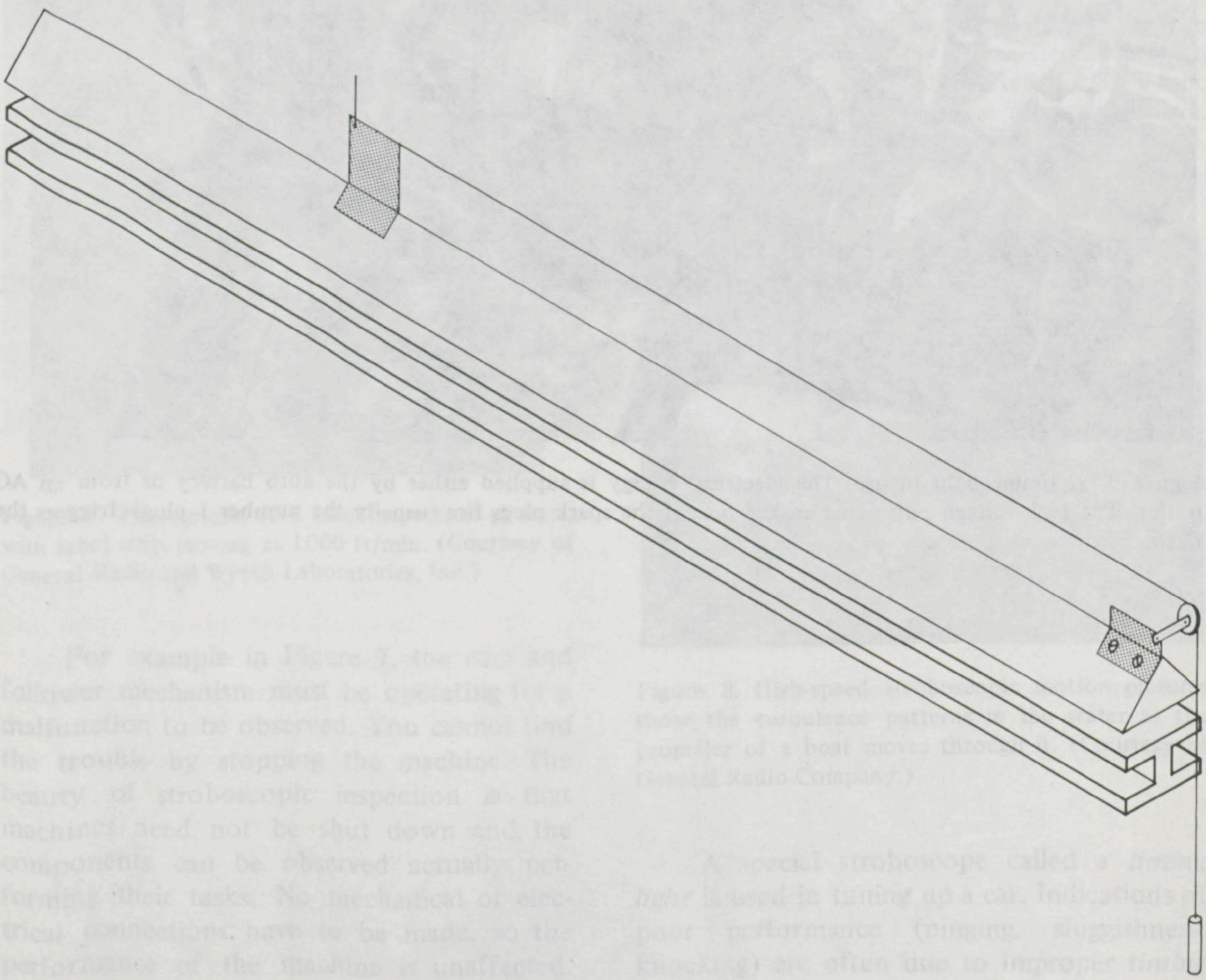


Figure 10.

b. Large glider, 10 g on the hanger

c. Large glider, 20 g on the hanger

4. Describe briefly what you observe.

5. Repeat the three cases, but this time get more information. Use a timer or stopwatch to measure how long it takes the glider to travel the length of the track. Record the time and the distance traveled for each case.

6. Compute the average velocity (v_{av}) for each case. *Average velocity (average speed)* is the distance traveled divided by the time of travel.*

7. Does the average velocity fully describe the motion, or is additional information needed? Does the speed of the glider change as it moves? The average velocity does not tell us how the velocity changes. One way to get additional information is to use a strobe.

8. Attach a white pointer to the glider (a piece of chalk taped to the end does very well). Illuminate the apparatus with the strobe and observe the motion for case a. The room should be darkened for the observation. Try different flash rates.

9. Briefly describe what you see. Make a rough sketch showing the positions of the glider for different strobe flashes. At best such a sketch will give you only a rough idea of what is happening, because you cannot observe and record the position of the glider at each flash. But the sketch can show general features, such as whether the distance between images increases or decreases.

10. Suppose that the strobe record of the motion is photographed by a camera. How can you analyze the information contained in such a photograph to obtain a more complete description of the motion?

*In physics *velocity* and *speed* mean somewhat different things, but we shall use them interchangeably in much of this module.

GRAPHS OF TRANSLATIONAL MOTION

A stroboscope and a camera provide a convenient way to record motion. Suppose that a camera is aimed at a dancer. The shutter is held open while the dancer moves across the field of view of the camera. In ordinary light, the photograph would show a smear along the path of the dancer. However, if the scene is illuminated with strobe light, the picture consists of a series of separate images, as shown in Figure 11. Each image shows the dancer's position when one of the flashes occurs. The distance between images corresponds to the distance the dancer moved between flashes of the strobe. A dark background is usually chosen so that the image of the moving object will not be "washed out" (i.e., so there is good contrast between the object and the background).

This procedure is a powerful way of providing detailed information about motion. For example, you may want to know how fast an object is moving during a particular part of its motion. This can be calculated from the strobe photo if the flash rate is known and if a meter stick is shown in the photo. However, the raw information is not in a convenient form, and the speed can be found more easily if the information about the motion is shown as a graph. How to make and interpret such graphs is the topic of this section.

Each of the strobe "photos" in Figure 12 provides a record of the motion of a glider on an air track. The strobe was set at a rate of one flash per second. Since there is one image every second, the numbered images can be used as a clock giving the time in seconds during the motion. The scale is included so that distances can be measured. Each of the photos matches one of the graphs in Figure 13, but not in the same order. On these graphs, time (t) is plotted along the horizontal axis and the distance traveled (x) along the vertical axis. Look at each of the photos and think carefully about what the corresponding graph should look like. *Before reading further, try to find a graph to match each of the photos.*

Our analysis starts with Figure 12B, because it is the simplest of the three. It is simple because the distance traveled between successive flashes is the same throughout the motion. Which of the graphs in Figure 13 has that characteristic? Look at graph A, which is repeated in Figure 14. Notice that, if we choose a particular time interval, the cor-

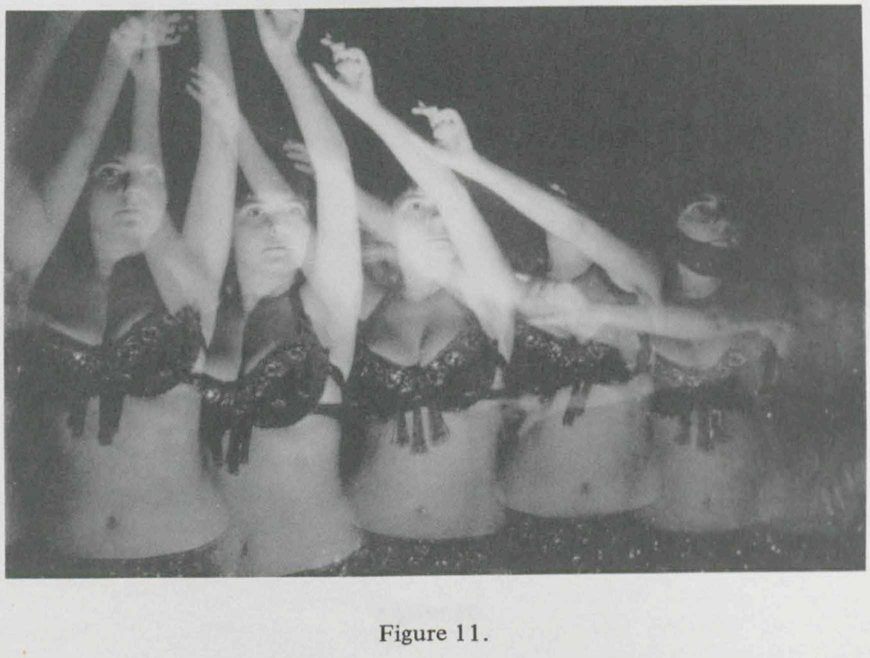


Figure 11.

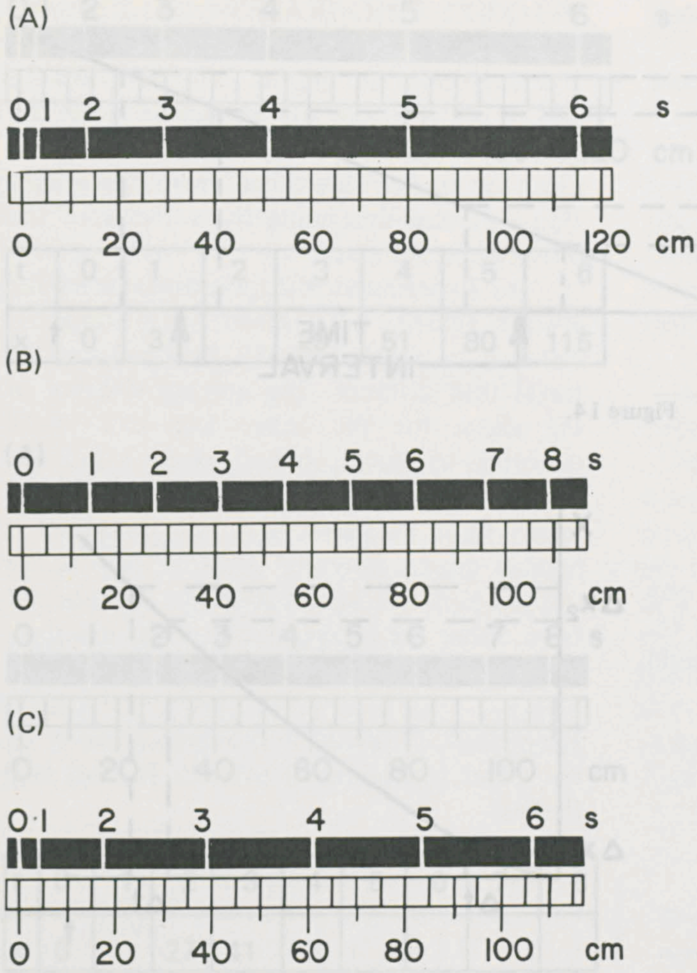


Figure 12. Strobe "photos" of a glider on an air track.

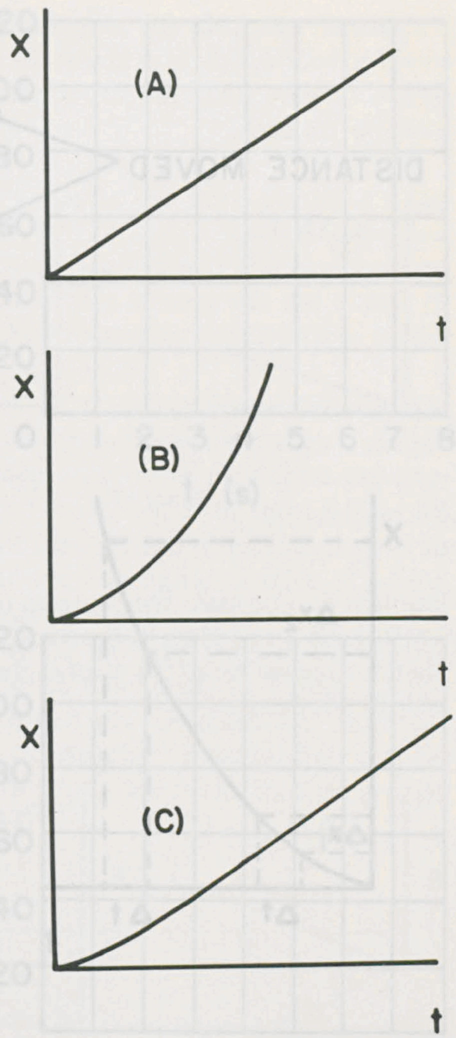


Figure 13.

responding increase in x does not depend on where on the time axis we begin the interval. The *change* in position (distance traveled) in a given time interval is constant. Thus, graph A does represent motion in which the distance traveled per unit time is the same throughout the motion. This is true whenever the position-time graph is a straight line.

Graph A matches photo B at least as far as general features are concerned. We will do a more careful comparison after matching general features of the remaining photos and graphs.

Photo A is slightly more complicated. The distance traveled between flashes increases as the glider moves along the air track. Which, if any, of the graphs has that feature? Look at graph B. This graph gets steeper as time passes. That is, the change in x corresponding to a given time interval gets larger as time passes.

To make talking about these graphs easier, we will use the convention that the Greek letter *delta* (Δ) means a *change* or an *interval* of some quantity. Thus Δt (pronounced "delta tee") might mean the time interval from one flash of the strobe to the next, and Δx would be the corresponding distance traveled in that interval.

To some extent, graph C is like graph B. The distance traveled during a time interval near the end (Δx_2) is greater than the distance traveled during an equal time interval near the beginning (Δx_1). But what happens if you compare the distance traveled during a

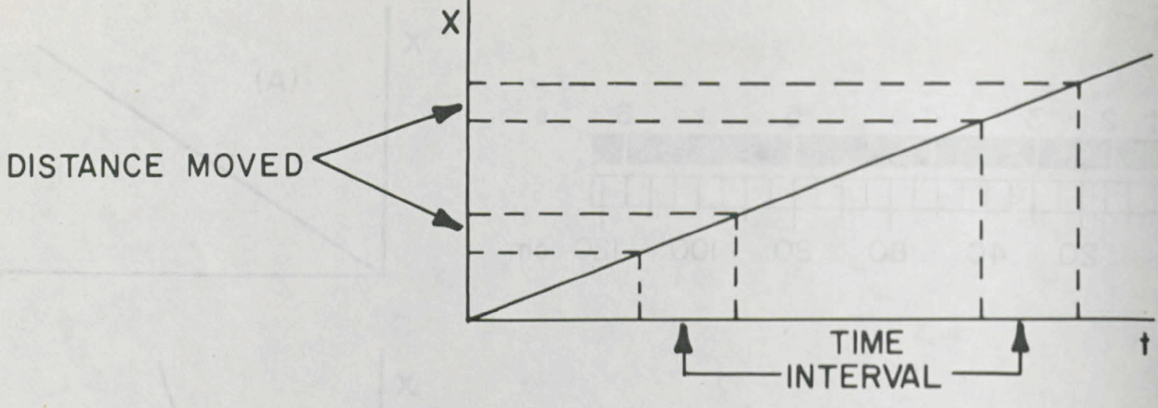


Figure 14.

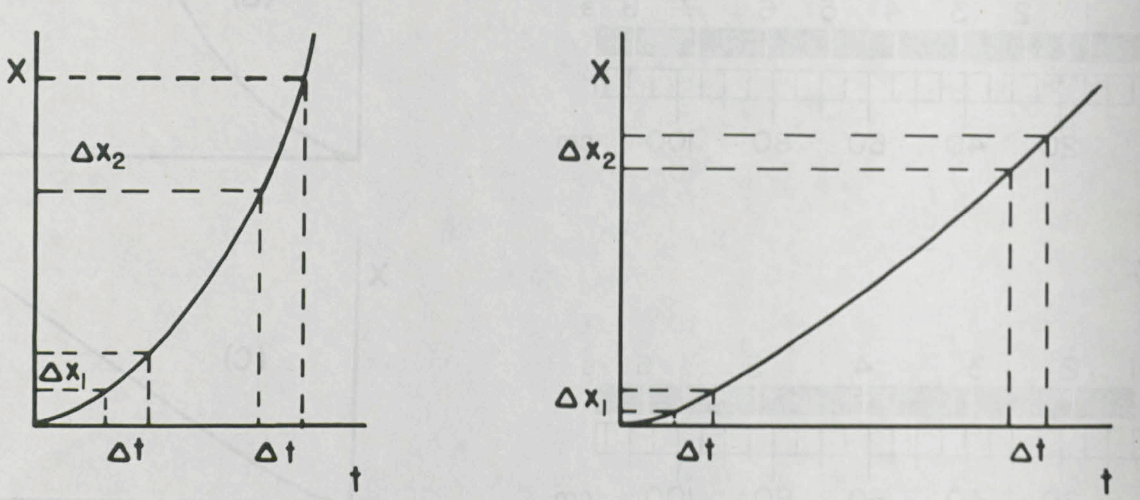


Figure 15.

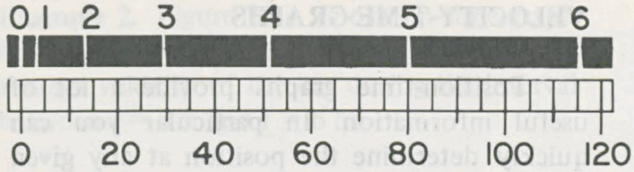
time interval at the end with one almost at the end?

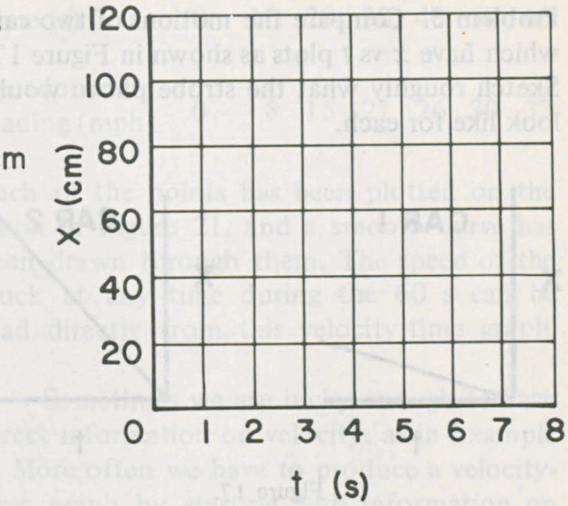
For graph C, near the end of motion the distance traveled in a given time is constant. That is why the graph is a straight line for large values of *t*. Notice that in photo C the images are equally spaced near the end. It seems that graph C matches photo C.

Now that the photos and graphs have been matched in a general way, we can study them more carefully. The photos are repeated in Figure 16. Tables are provided for recording the location of the glider at each flash of the strobe light. Graphs are provided for plotting the points. Some of each table has been filled in.

Problem 4. Finish the tables in Figure 16, then plot the points. Draw a smooth curve through each set of points. How do these graphs compare with those in Figure 13?

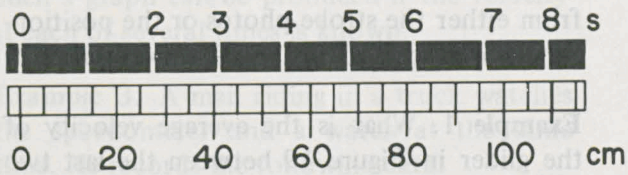
Even though you have constructed the graphs from actual data, you must be careful when you assume that the curves accurately describe the motion. All one can tell from the photos is the location of the glider at a few particular times. The photo gives no information about where the glider is at other times. When we draw a smooth curve connecting the points, we are assuming that the glider moves "smoothly" during the intervals between flashes. This assumption is usually a good one, and it enables us to use the graph to determine the position of the glider between flashes. In cases where there is doubt about the smoothness of the motion, the experiment can be repeated using a higher flash rate. A higher flash rate provides more images and therefore more points to plot. The additional detail should reveal any deviations from smooth motion.

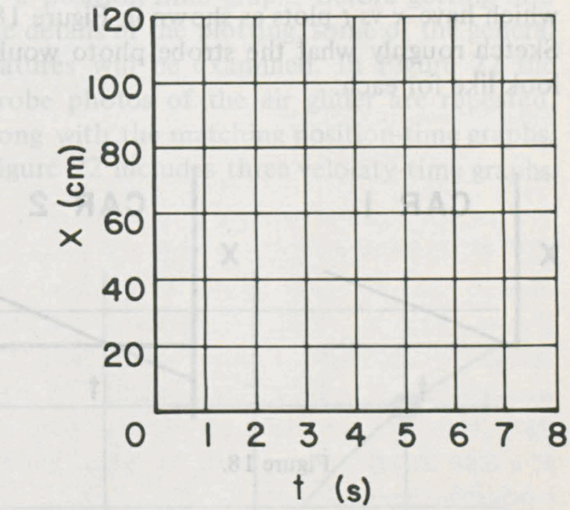




t	0	1	2	3	4	5	6
x	0	3		29	51	80	115

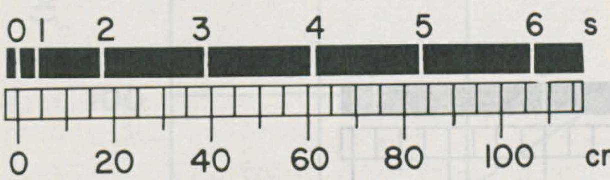
(A)

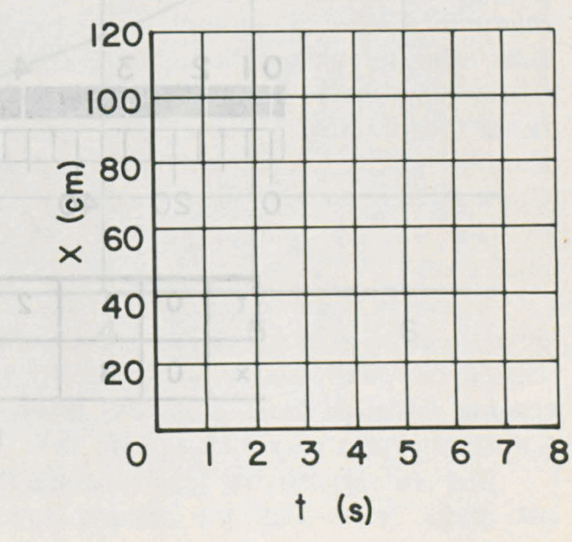




t	0	1	2	3	4	5	6	7	8
x	0		27	41					

(B)





t	0	1	2	3	4	5	6
x	0						

(C)

Figure 16.

Problem 5. Compare the motions of two cars which have x vs t plots as shown in Figure 17. Sketch roughly what the strobe photo would look like for each.

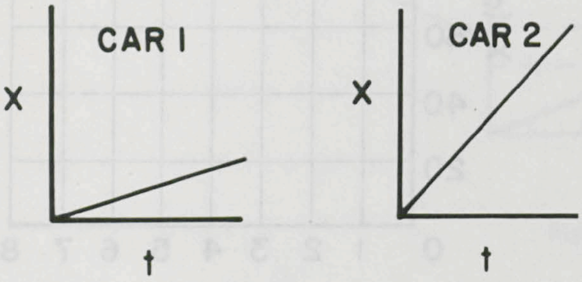


Figure 17.

Problem 6. Compare the motions of two cars which have x vs t plots as shown in Figure 18. Sketch roughly what the strobe photo would look like for each.

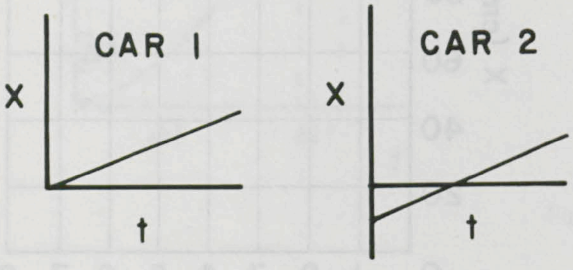


Figure 18.

VELOCITY-TIME GRAPHS

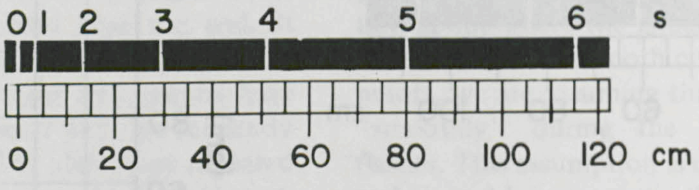
Position-time graphs provide a lot of useful information. In particular you can quickly determine the position at any given time by reading the value from the graph. You can also determine the velocity of the object from the graph.

In Experiment A-2, you determined the average velocity of a glider by dividing the distance it traveled by the time required to travel that distance. The average velocity of the glider for the entire trip is a useful quantity to know, but it doesn't tell the whole story of the motion. For example, one might want to know how fast the glider is moving during different parts of its trip. We can see how this information can be obtained from either the strobe photos or the position-time graphs, by looking at a few examples.

Example 1. What is the average velocity of the glider in Figure 19 between the last two flashes?

Solution. During the interval between the last two flashes the glider moves from $x = 80$ cm to $x = 115$ cm, a distance of $\Delta x = 35$ cm. The time interval between flashes Δt is one second. Therefore the average velocity is

$$v_{\mathrm{av}} = \frac{\Delta x}{\Delta t} = \frac{35\ \mathrm{cm}}{1\ \mathrm{s}} = 35\ \mathrm{cm/s}$$



t	0	1	2	3	4	5	6
x	0	3		29	51	80	115

Figure 19.

Example 2. Figure 20 is a position-time graph for a car moving along a highway. What is the average velocity of the car during the interval between $t = 2$ h and $t = 6$ h?

Solution. During the specified time interval, the value of x changes from 50 km to 250 km. The car moves 200 km in 4 h. Therefore, the average velocity is:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{200 \text{ km}}{4 \text{ h}} = 50 \text{ km/h}$$

(Since 1 km = 0.62 mi, this is about 31 mph.)

When velocity is of major interest, it is convenient to make a velocity-time graph. Such a graph can be produced if the velocity at each of several times is known.

Example 3. A man riding in a truck watches the speedometer and a watch at the same time. He records the following data:

Time (s)	0	10	20	30	40	50	60
Speedometer reading (mph)	0	8	15	21	26	30	33

Each of the points has been plotted on the graph in Figure 21, and a smooth curve has been drawn through them. The speed of the truck at any time during the 60 s can be read directly from this velocity-time graph.

Sometimes we are lucky enough to have direct information on velocity, as in Example 3. More often we have to produce a velocity-time graph by starting with information on position and time. This information may be in the form of a strobe photo, a table of values, or a position-time graph. Before getting into the details of the plotting, some of the general features will be examined. In Figure 22 the strobe photos of the air glider are repeated, along with the matching position-time graphs. Figure 22 includes three velocity-time graphs.

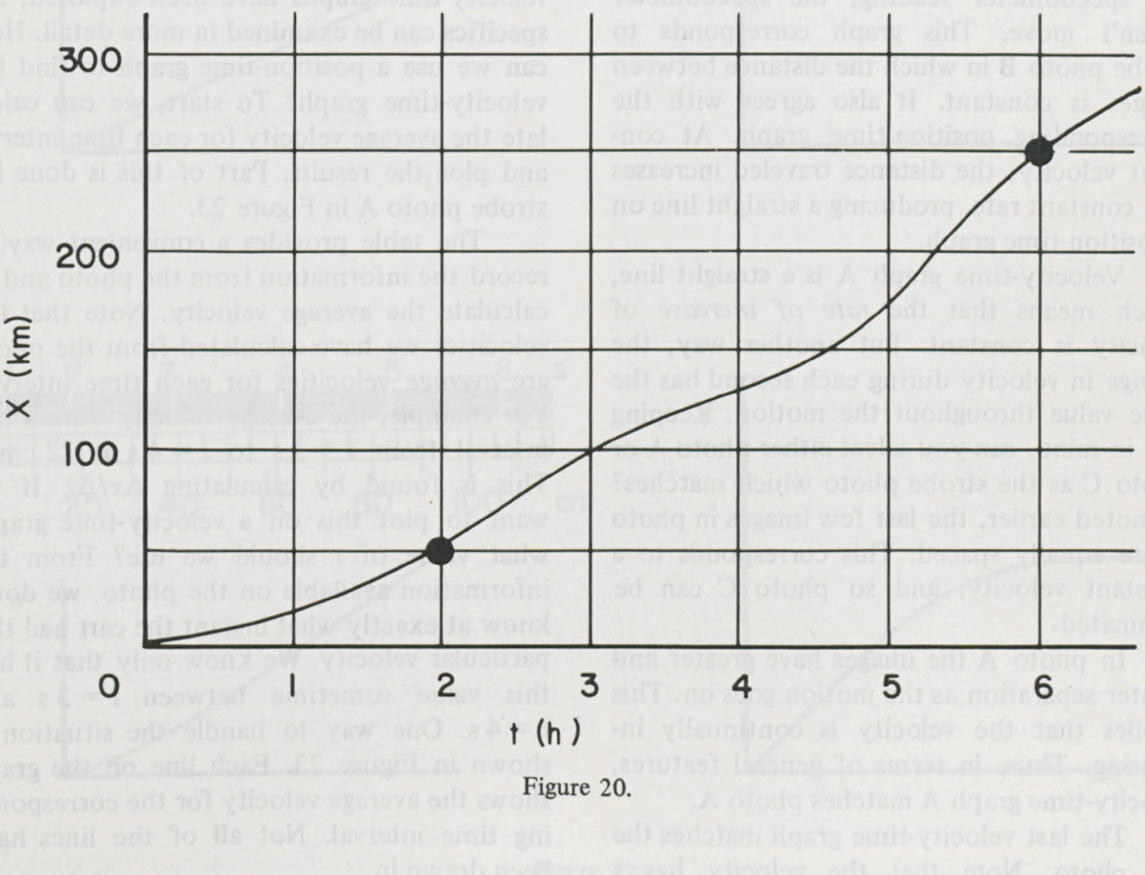


Figure 20.

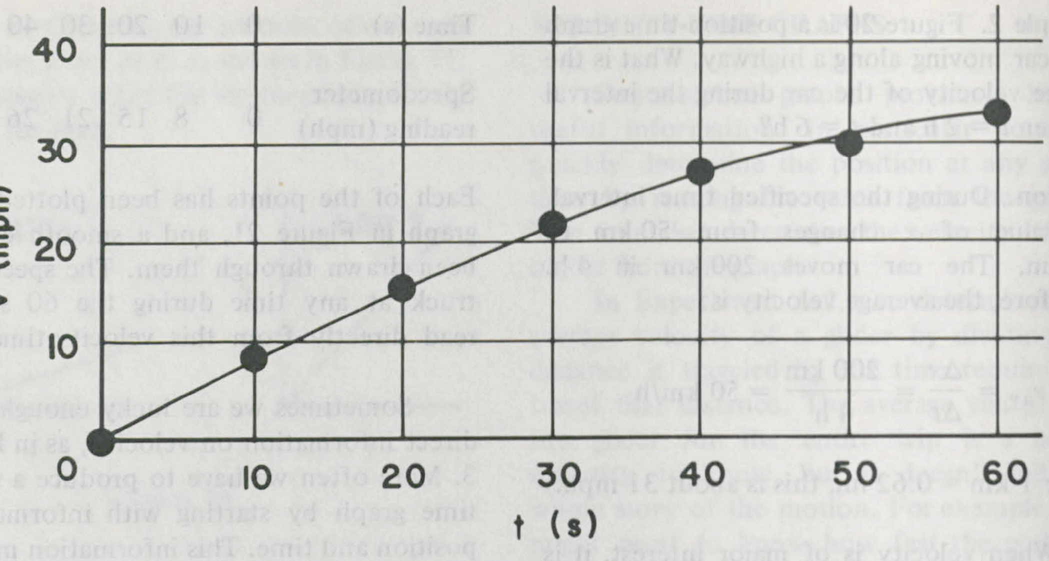


Figure 21

Each one of the velocity-time graphs matches one of the strobe photos. *Try to match them before reading further.*

The velocity-time graph B in Figure 22 is fairly simple. It represents motion in which the velocity is constant; if you think of it as the speedometer reading, the speedometer doesn't move. This graph corresponds to strobe photo B in which the distance between images is constant. It also agrees with the corresponding position-time graph. At constant velocity, the distance traveled increases at a constant rate, producing a straight line on a position-time graph.

Velocity-time graph A is a straight line, which means that the *rate of increase* of velocity is constant. Put another way, the *change* in velocity during each second has the same value throughout the motion. Keeping this in mind, can you select either photo A or photo C as the strobe photo which matches? As noted earlier, the last few images in photo C are equally spaced. This corresponds to a constant velocity, and so photo C can be eliminated.

In photo A the images have greater and greater separation as the motion goes on. This implies that the velocity is continually increasing. Thus, in terms of general features, velocity-time graph A matches photo A.

The last velocity-time graph matches the last photo. Note that the velocity has a constant value near the end of the motion. There the images are equally spaced, the position-time graph becomes a straight line, and the velocity-time graph becomes a horizontal straight line.

Now that the general features of the velocity-time graphs have been explored, the specifics can be examined in more detail. How can we use a position-time graph to find the velocity-time graph? To start, we can calculate the average velocity for each time interval and plot the results. Part of this is done for strobe photo A in Figure 23.

The table provides a convenient way to record the information from the photo and to calculate the average velocity. Note that the velocities we have calculated from the photo are *average* velocities for each time interval. For example, the average velocity during the interval from $t = 3$ s to $t = 4$ s is 22 cm/s. This is found by calculating $\Delta x/\Delta t$. If we want to plot this on a velocity-time graph, what value of t should we use? From the information available on the photo, we don't know at exactly what instant the cart had this particular velocity. We know only that it had this value sometime between $t = 3$ s and $t = 4$ s. One way to handle the situation is shown in Figure 23. Each line on the graph shows the average velocity for the corresponding time interval. Not all of the lines have been drawn in.

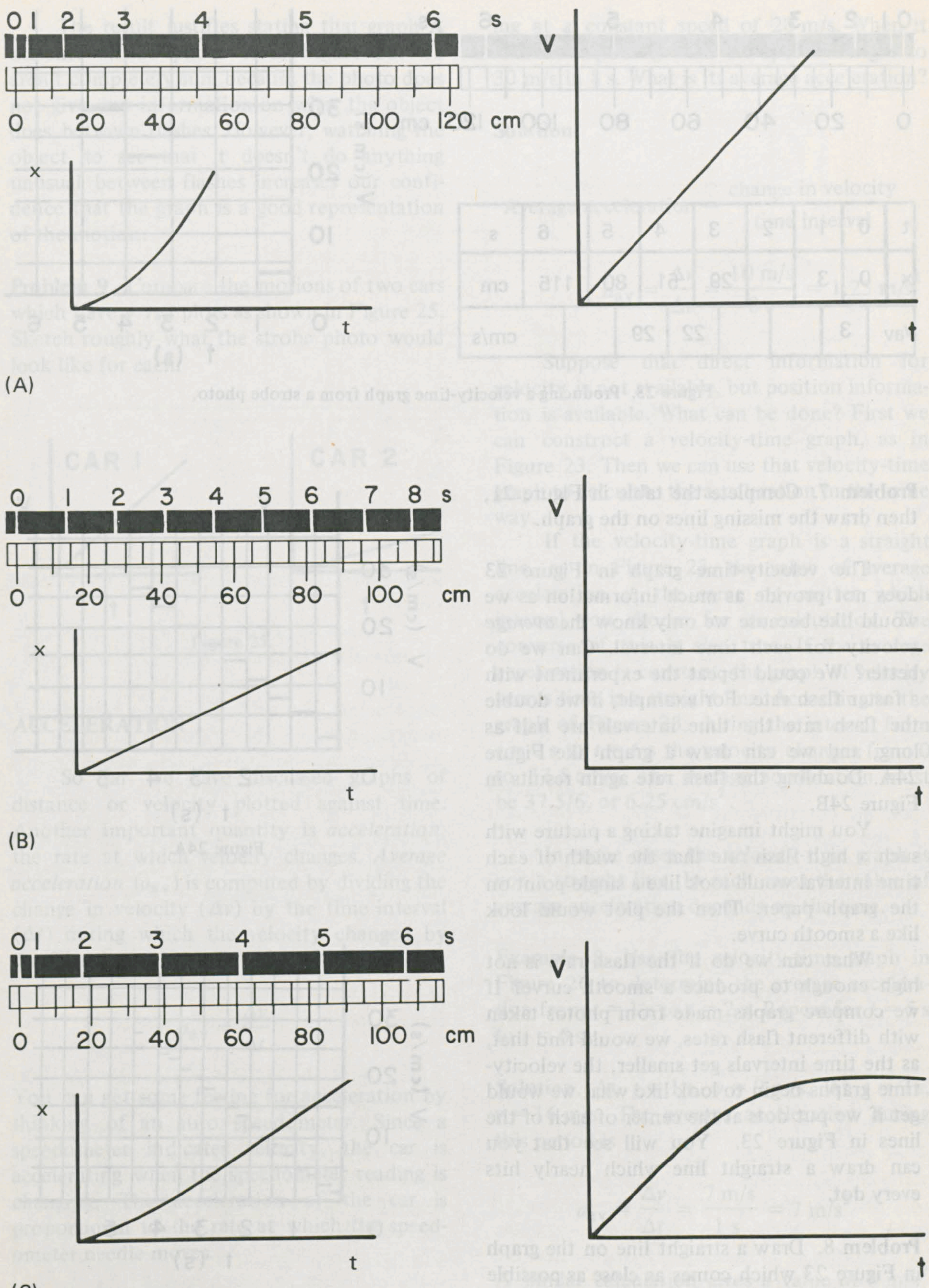
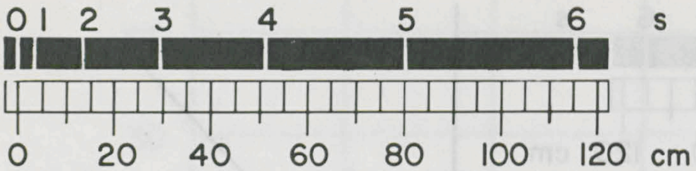

0 1 2 3 4 5 6 s
0 20 40 60 80 100 120 cm
x
t
V
t
(A)
0 1 2 3 4 5 6 7 8 s
0 20 40 60 80 100 cm
x
t
V
t
(B)
0 1 2 3 4 5 6 s
0 20 40 60 80 100 cm
x
t
V
t
(C)

Figure 22.



t	0	1	2	3	4	5	6	s
x	0	3		29	51	80	115	cm
Vav	3			22	29			cm/s

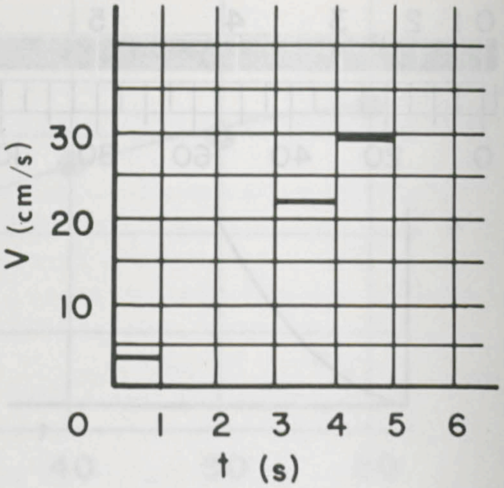


Figure 23. Producing a velocity-time graph from a strobe photo.

Problem 7. Complete the table in Figure 21, then draw the missing lines on the graph.

The velocity-time graph in Figure 23 does not provide as much information as we would like because we only know the *average* velocity for each time interval. Can we do better? We could repeat the experiment with a faster flash rate. For example, if we double the flash rate the time intervals are half as long, and we can draw a graph like Figure 24A. Doubling the flash rate again results in Figure 24B.

You might imagine taking a picture with such a high flash rate that the width of each time interval would look like a single point on the graph paper. Then the plot would look like a smooth curve.

What can we do if the flash rate is not high enough to produce a smooth curve? If we compare graphs made from photos taken with different flash rates, we would find that, as the time intervals get smaller, the velocity-time graphs begin to look like what we would get if we put dots at the center of each of the lines in Figure 23. You will see that you can draw a straight line which nearly hits every dot.

Problem 8. Draw a straight line on the graph in Figure 23 which comes as close as possible to all the dots.

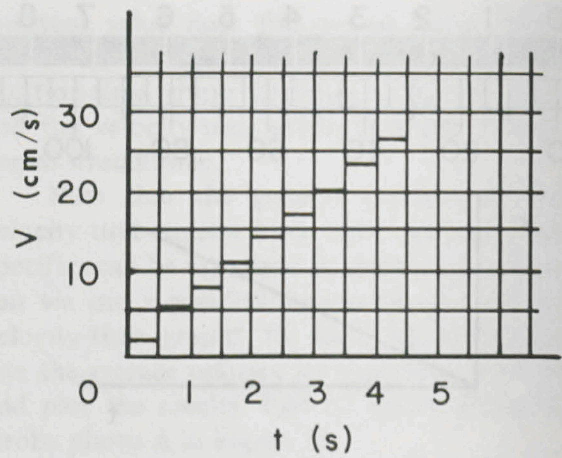


Figure 24A.

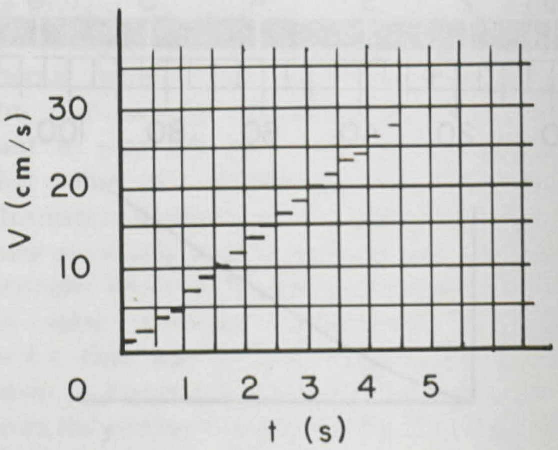


Figure 24B.

The result justifies stating that graph A matches strobe photo A in Figure 22. We aren't completely sure because the photo does not give any information on what the object does between flashes. However, watching the object to see that it doesn't do anything unusual between flashes increases our confidence that the graph is a good representation of the motion.

Problem 9. Compare the motions of two cars which have v vs t plots as shown in Figure 25. Sketch roughly what the strobe photo would look like for each.

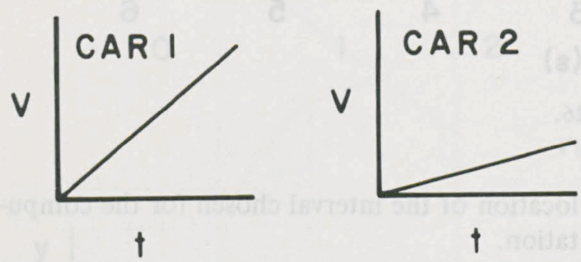


Figure 25.

ACCELERATION

So far we have discussed graphs of distance or velocity plotted against time. Another important quantity is *acceleration,* the rate at which velocity changes. *Average acceleration* (a_{av}) is computed by dividing the change in velocity (Δv) by the time interval (Δt) during which the velocity changed by Δv. That is:

$$a_{av} = \frac{\Delta v}{\Delta t}$$

You can get some feeling for acceleration by thinking of an auto speedometer. Since a speedometer indicates velocity, the car is accelerating when the speedometer reading is changing. The acceleration of the car is proportional to the rate at which the speedometer needle moves.

Example 4. Following a truck, a car is moving at a constant speed of 20 m/s. When it reaches a passing zone its speed changes to 30 m/s in 8 s. What is its average acceleration?

Solution.

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{10 \text{ m/s}}{8 \text{ s}} = 1.25 \text{ m/s}^2$$

Suppose that direct information for velocity is not available, but position information is available. What can be done? First we can construct a velocity-time graph, as in Figure 23. Then we can use that velocity-time graph to calculate the acceleration in the same way.

If the velocity-time graph is a straight line, as in Figure 23, the value of average acceleration is the same no matter which interval you choose for its calculation. The converse of this is also true. If the average acceleration is constant, the graph of velocity versus time is a straight line. According to the graph of Figure 23, during the interval from $t = 0$ s to $t = 6$ s the velocity changes from 0 to 37.5 cm/s. The average acceleration must be 37.5/6, or 6.25 cm/s^2.

In some cases the velocity-time graph is *not* a straight line. In such cases the value of average acceleration depends on the time.

Example 5. Use the velocity-time graph in Figure 26 to determine the average acceleration from $t = 1$ s to $t = 2$ s. Repeat for $t = 5$ s to $t = 6$ s.

Solution. At $t = 1$ s, $v = 9$ m/s. At $t = 2$ s, $v = 16$ m/s. The average acceleration during this period is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{7 \text{ m/s}}{1 \text{ s}} = 7 \text{ m/s}^2$$

A similar calculation gives a value of 0 m/s^2 for the average acceleration between $t = 5$ s

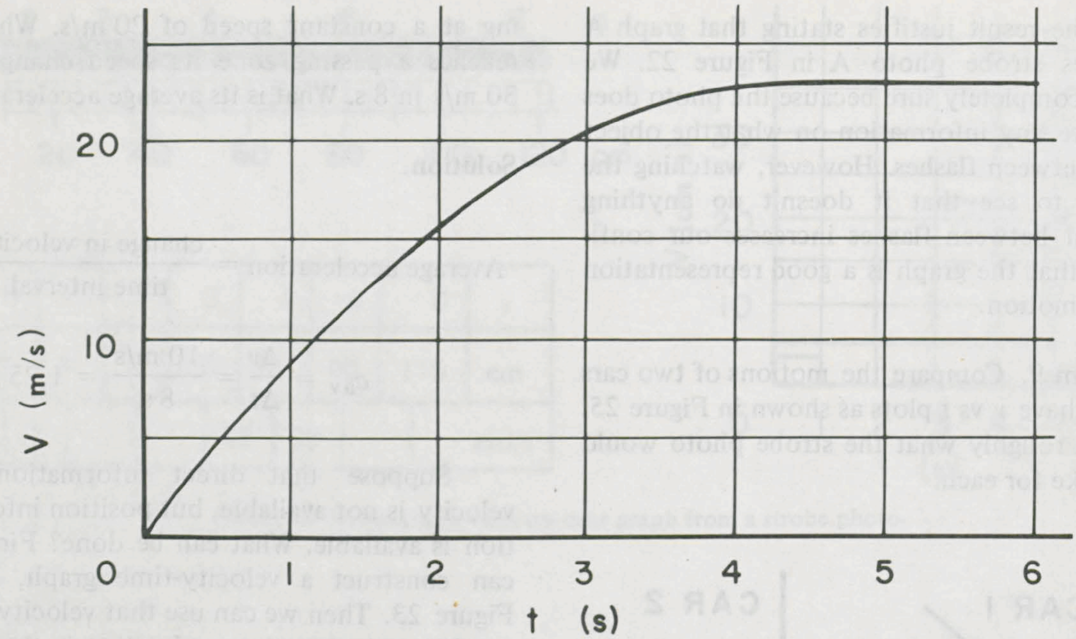


Figure 26.

and $t = 6$ s. This is because the velocity is not changing on this part of the graph.

Continuing this process for each 1-s interval, we can draw an acceleration-time graph, such as Figure 27.

SLOPE

The "steepness" of a straight-line graph can be described by its *slope*. The slope is the *change* in the vertical coordinate (often called the *rise*) divided by the corresponding *change* in the horizontal coordinate (often called the *run*). For a straight-line graph, the slope is the same everywhere. If we call the coordinates of the ends of the horizontal interval x_1 and x_2, and the coordinates of the ends of the corresponding vertical interval y_1 and y_2, as shown in Figure 28A, then the slope m is given by

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \qquad (1)$$

Figure 28A is redrawn in Figure 28B with a different interval chosen for the computation of its slope. Notice that the calculated value of the slope of a straight-line graph does not depend on the size or the location of the interval chosen for the computation.

The slope can be positive, negative, zero, or infinite (undefined). When y increases as x increases, the slope is *positive.* A *negative* slope means that y decreases as x increases. A *zero* slope means that y does not change as x increases. An *infinite* slope means that y changes without any change of x.

Question 2. Determine whether the slopes of the graphs shown in Figure 29 are positive, negative, zero, or infinite.

Question 3. How is the slope of a position-time graph related to the average velocity? How is the slope of a velocity-time graph related to the average acceleration?

You might wonder if the idea of slope has meaning for curved-line graphs. Clearly the "steepness" of such a graph changes from point to point along the line. However, if we can approximate a small part of the graph by a straight line, the slope of a curved line can be defined. Figure 30 illustrates the nature of the problem. The largest interval leads to a value of the slope which is really the slope of the straight line connecting points A and D. It

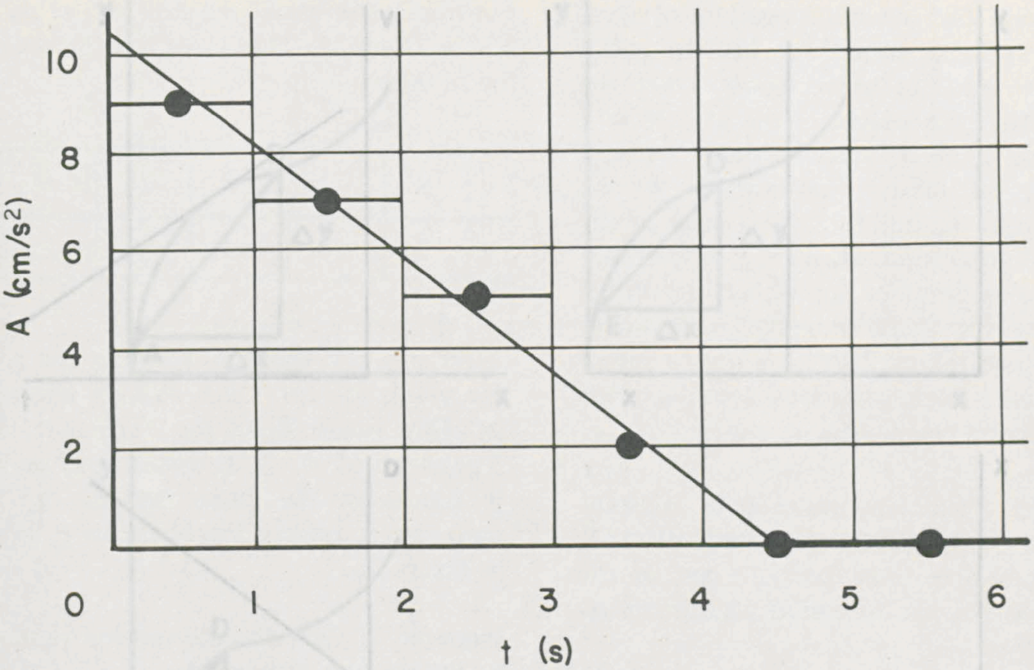


Figure 27.

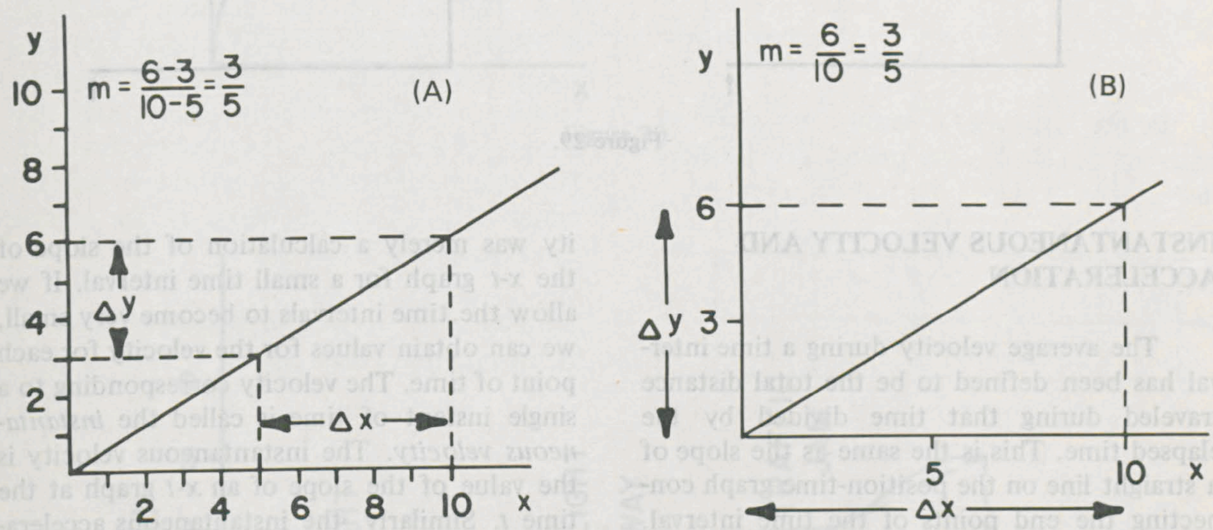


Figure 28.

is an average value of the slope of the graph. Choosing a smaller interval allows calculation of a different average slope—that of the straight line between C and D. A still smaller interval allows calculation of another average value of the slope between E and D. We might imagine that we could choose ever-smaller intervals and that each such interval would lead to a value of the slope which is a closer approximation to the actual slope *at a given point* on the graph. In fact, a mathematical procedure for doing this exists, but it is beyond the scope of this module. The slope at a point on the curve is the same as the slope of the straight line drawn *tangent* to the curve through the point. The tangent line touches the curve at only a single point. A procedure for constructing the tangent exists, and one can know the slope of a curved graph at every point on the graph.

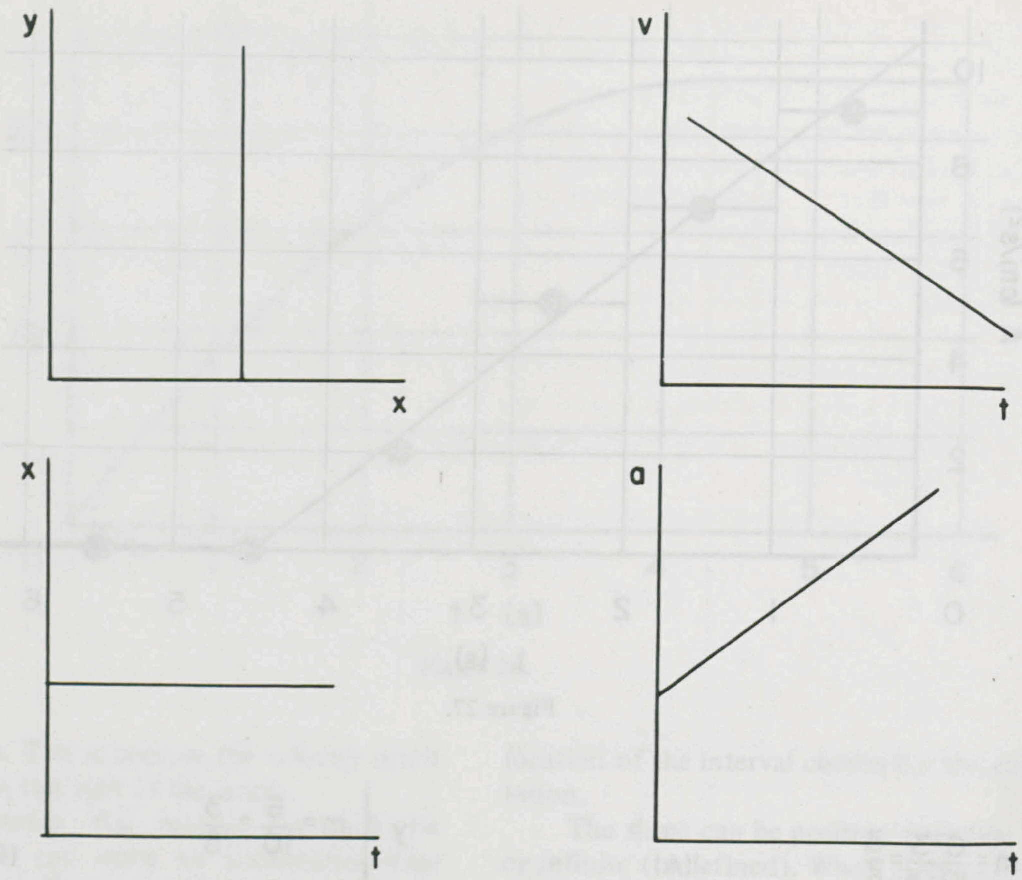


Figure 29.

INSTANTANEOUS VELOCITY AND ACCELERATION

The average velocity during a time interval has been defined to be the total distance traveled during that time divided by the elapsed time. This is the same as the slope of a straight line on the position-time graph connecting the end points of the time interval. Figure 31 is a graph of position and time for a car making a five-mile trip through city traffic. Although the average speed for the trip is computed to be 15 mph, it is clear from the graph that the car did not travel at a constant velocity of 15 mph for the entire trip.

By looking at smaller time intervals, we can compute values of the slope (average velocity), and this allows us to characterize more accurately the motion of the car. This is precisely what was done graphically in Figures 23 and 24. Each calculation of average velocity was merely a calculation of the slope of the *x-t* graph for a small time interval. If we allow the time intervals to become very small, we can obtain values for the velocity for each point of time. The velocity corresponding to a single instant of time is called the *instantaneous velocity.* The instantaneous velocity is the value of the slope of an *x-t* graph at the time *t.* Similarly, the instantaneous acceleration is the value of the slope of a velocity-time graph at the time of interest.

In this module, you will usually need only to be able to find the instantaneous velocity or acceleration for straight-line graphs, which have constant slopes.

INITIAL VALUES FOR POSITION AND VELOCITY

The values of position and velocity when the clock is turned on ($t = 0$) are called the *initial values.* The cases we have discussed

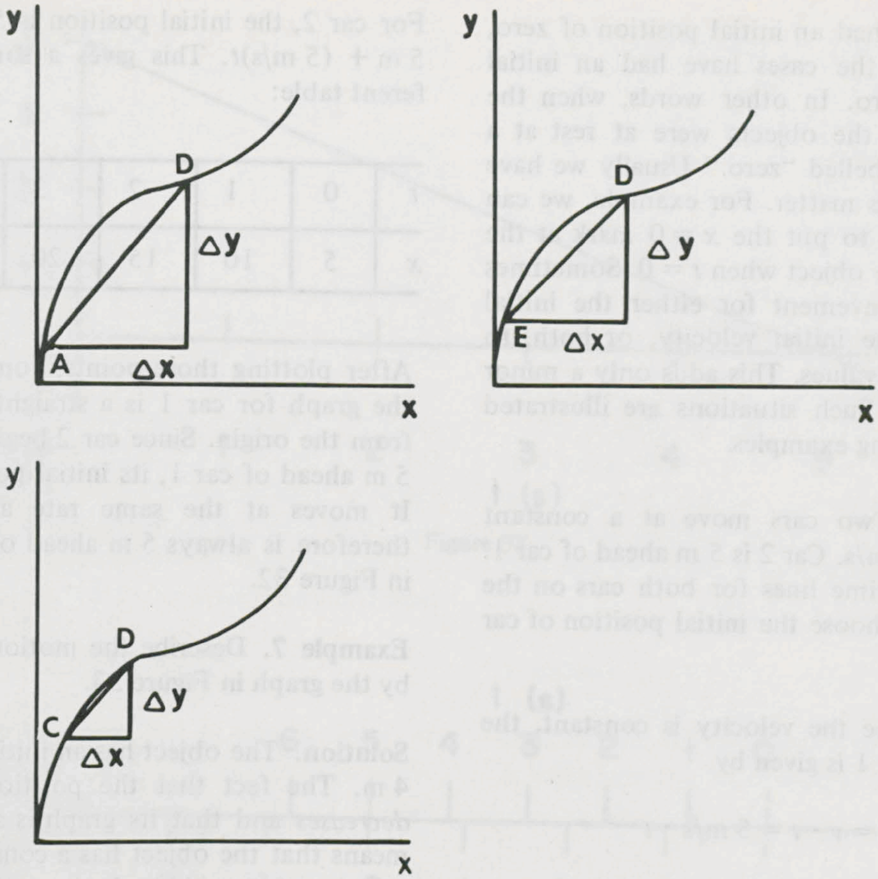


Figure 30.

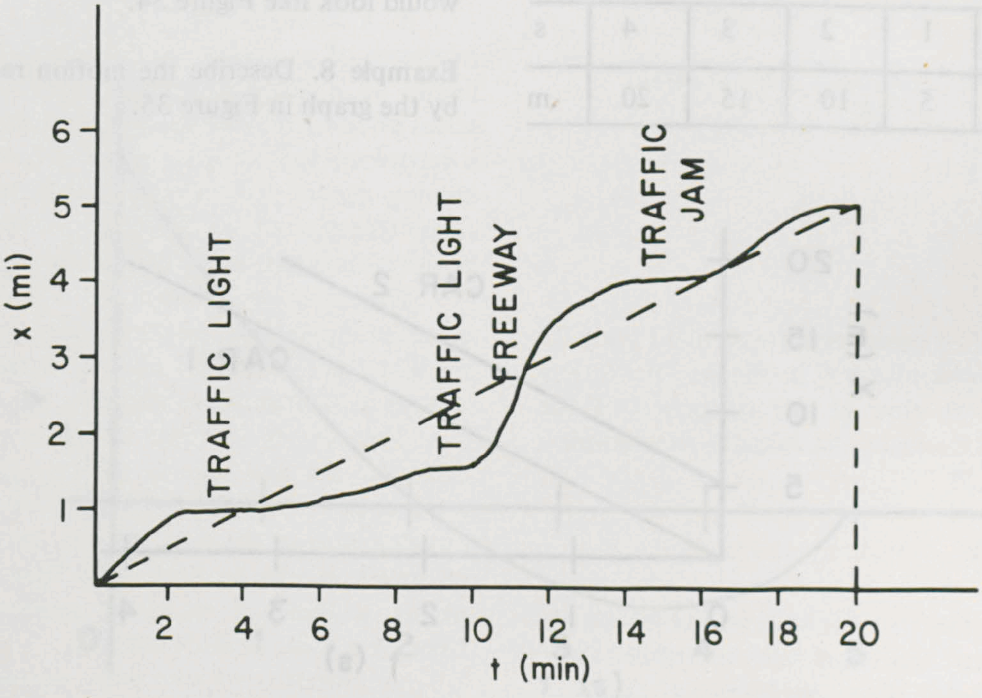


Figure 31.

thus far have had an initial position of zero, and most of the cases have had an initial velocity of zero. In other words, when the clock started, the objects were at rest at a position we labelled "zero." Usually we have a choice in this matter. For example, we can always choose to put the $x = 0$ mark at the location of the object when $t = 0$. Sometimes it is more convenient for either the initial position or the initial velocity, or both, to have non-zero values. This adds only a minor complication. Such situations are illustrated by the following examples.

Example 6. Two cars move at a constant velocity of 5 m/s. Car 2 is 5 m ahead of car 1. Plot position-time lines for both cars on the same graph. Choose the initial position of car 1 to be $x = 0$.

Solution. Since the velocity is constant, the position of car 1 is given by

$$x = v \cdot t = 5 \text{ m/s} \cdot t$$

Before drawing a graph, we need to construct a table of data points. For car 1 we have

t	0	1	2	3	4	s
x	0	5	10	15	20	m

For car 2, the initial position is 5 m and $x = 5 \text{ m} + (5 \text{ m/s})t$. This gives a somewhat different table:

t	0	1	2	3	4	s
x	5	10	15	20	25	m

After plotting those points, you can see that the graph for car 1 is a straight line starting from the origin. Since car 2 begins its motion 5 m ahead of car 1, its initial position is 5 m. It moves at the same rate as car 1 and therefore is always 5 m ahead of it as shown in Figure 32.

Example 7. Describe the motion represented by the graph in Figure 33.

Solution. The object has an initial position of 4 m. The fact that the position coordinate *decreases* and that its graph is a straight line means that the object has a constant *negative* velocity. A negative velocity means only that the object is moving in the negative direction. The object continues past $x = 0$ into negative values of x. A strobe photo of the motion would look like Figure 34.

Example 8. Describe the motion represented by the graph in Figure 35.

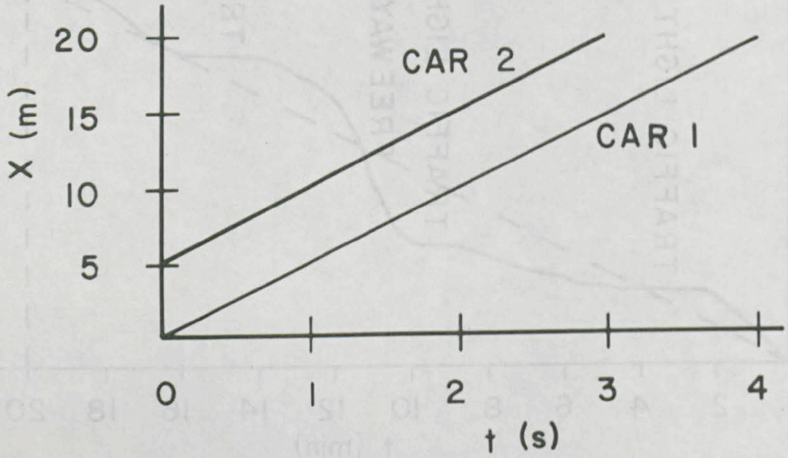


Figure 32.

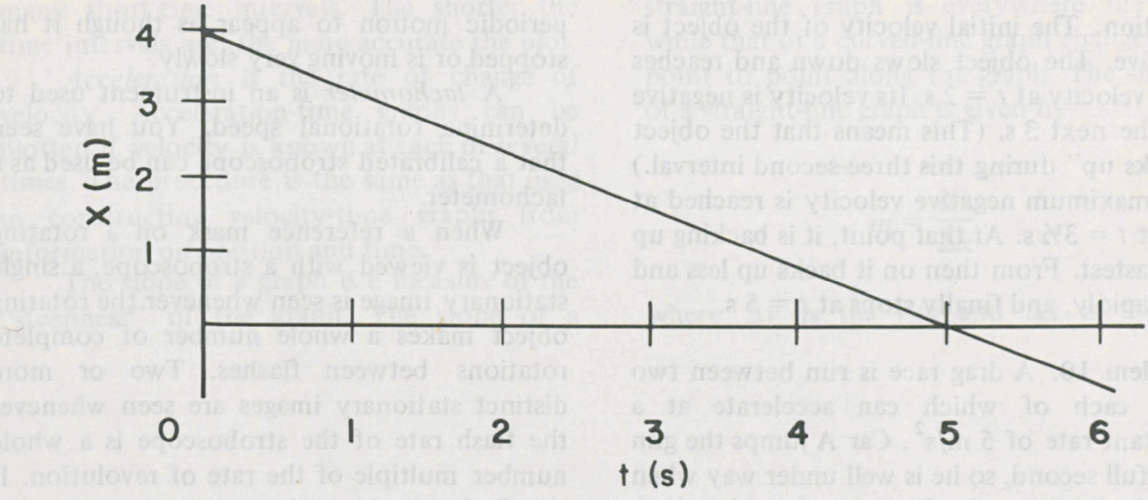


Figure 33.

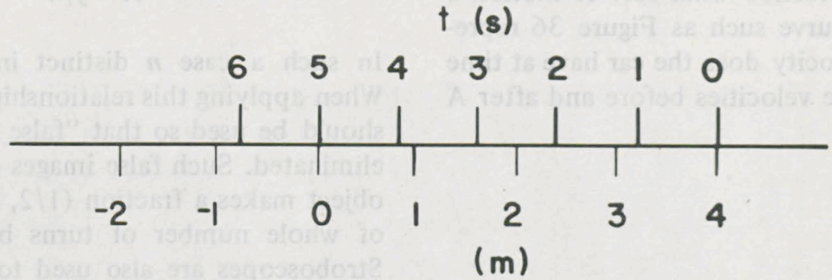


Figure 34.

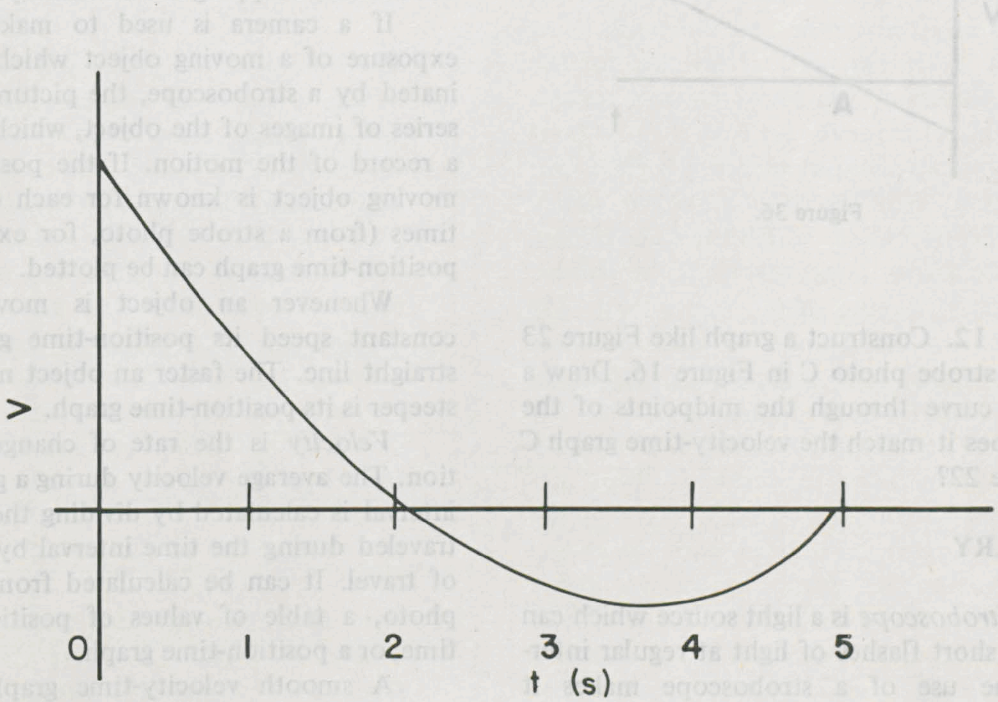


Figure 35.

Solution. The initial velocity of the object is positive. The object slows down and reaches zero velocity at $t = 2$ s. Its velocity is negative for the next 3 s. (This means that the object "backs up" during this three-second interval.) The maximum negative velocity is reached at about $t = 3\frac{1}{2}$ s. At that point, it is backing up the fastest. From then on it backs up less and less rapidly, and finally stops at $t = 5$ s.

Problem 10. A drag race is run between two cars, each of which can accelerate at a constant rate of 5 m/s^2. Car A jumps the gun by a full second, so he is well under way when the clock starts. Car B starts when the clock starts. Sketch velocity-time plots for the two cars.

Problem 11. Describe what sort of motion a velocity-time curve such as Figure 36 represents. What velocity does the car have at time A? How do the velocities before and after A differ?

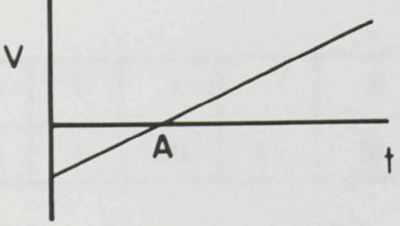


Figure 36.

Problem 12. Construct a graph like Figure 23 but for strobe photo C in Figure 16. Draw a smooth curve through the midpoints of the lines. Does it match the velocity-time graph C in Figure 22?

SUMMARY

A *stroboscope* is a light source which can provide short flashes of light at regular intervals. The use of a stroboscope makes it possible for any rotational motion or other periodic motion to appear as though it has stopped or is moving very slowly.

A *tachometer* is an instrument used to determine rotational speed. You have seen that a calibrated stroboscope can be used as a tachometer.

When a reference mark on a rotating object is viewed with a stroboscope, a single stationary image is seen whenever the rotating object makes a whole number of complete rotations between flashes. Two or more distinct stationary images are seen whenever the flash rate of the stroboscope is a whole number multiple of the rate of revolution. If the flash rate f is n times as great as the rate of revolution of the object R, then the three quantities are related by the equation

$$R = f/n$$

In such a case n distinct images are seen. When applying this relationship a table always should be used so that "false images" can be eliminated. Such false images occur when the object makes a fraction (1/2, 1/3, 1-1/2, etc.) of whole number of turns between flashes. Stroboscopes are also used to "freeze" periodic motion, and inspect moving parts without actually stopping the machinery.

If a camera is used to make a time exposure of a moving object which is illuminated by a stroboscope, the picture shows a series of images of the object, which provides a record of the motion. If the position of a moving object is known for each of several times (from a strobe photo, for example), a position-time graph can be plotted.

Whenever an object is moving at a constant speed its position-time graph is a straight line. The faster an object moves, the steeper is its position-time graph.

Velocity is the rate of change of position. The average velocity during a given time interval is calculated by dividing the distance traveled during the time interval by the time of travel. It can be calculated from a strobe photo, a table of values of position versus time, or a position-time graph.

A smooth velocity-time graph can be drawn if the average velocity is known for

many short time intervals. The shorter the time intervals are, the more accurate the plot.

Acceleration is the rate of change of velocity. Acceleration-time graphs can be plotted if velocity is known at each of several times. The procedure is the same as that used in constructing velocity-time graphs from information on position and time.

The slope of a graph is a measure of the "steepness" of the graph. The slope of a straight-line graph is everywhere the same while that of a curved-line graph changes from point to point along the graph. The slope m of a straight-line graph is given by

$$m = \frac{\Delta y}{\Delta x}$$

where Δy is the rise and Δx is the run.

SECTION B

A Mathematical Approach to Kinematics

In Section A, motion was described by the use of graphs. In Section B, the graphical study of motion will be extended to include basic mathematical descriptions. In this section you will continue to study how objects move. The description of motion is called *kinematics.* The study of why objects move as they do, called *dynamics,* begins in Section C of the module. You can continue your study of kinematics by observing a common example of accelerated motion.

EXPERIMENT B-1. Using the Stroboscope to Study Translational Motion

In this experiment a stroboscope is used to record on film the motions of a glider on an air track and of a freely falling ball. The information contained in these strobe photographs is displayed in tables and then in graphical form. You will look for mathematical relationships among the quantities involved.

Procedure

1. Set up the Polaroid camera and a level air track as in Figure 37. The track is level if the glider does not move when released at any point on the track. (For most air tracks, perfect leveling for the full length of the track requires considerable adjustment. Get it as close to level as you can without spending a great deal of time.) With the room darkened, take a picture of a glider which has been given a gentle shove along the track. A pointer attached to the glider (see Figure 38) simplifies the process of measurement. You should have some kind of reference scale, such as a meter stick, in the picture. See your instructor if you do not know how to make a "time exposure" with your camera. Turn on the strobe, open the shutter, push the glider, and close the shutter when the glider strikes the end of the track. A flash rate of 300 fpm (5 flashes per second) gives a convenient time interval and an adequate number of images. Record the flash rate. Figure 38 is a typical strobe photo of a glider taken with a flash rate of 300 fpm.

2. Using the same procedure make another strobe photograph, either of a glider released from the end of a tilted air track, or of a falling ball. A flash rate of 600 fpm should be adequate for the glider, and a flash rate of 1200 fpm is best for the falling ball. Typical results are illustrated in the analysis which follows.

3. The images of the glider and ball should be sharp and clear because each stroboscope flash lasts only about one-millionth of a second. The object moves such a short distance in one-millionth of a second that its image is not blurred. The position of the glider and ball at the time of each flash can be measured, making use of the reference scale in each photograph. Use the first clear image as the zero position and measure the position x of the ball for as many successive flashes of the strobe as you can. Each position should be measured from the same zero point on the photo.

4. For each photograph, make a table similar to Table II. Using the known flash rate of the strobe, and the distances

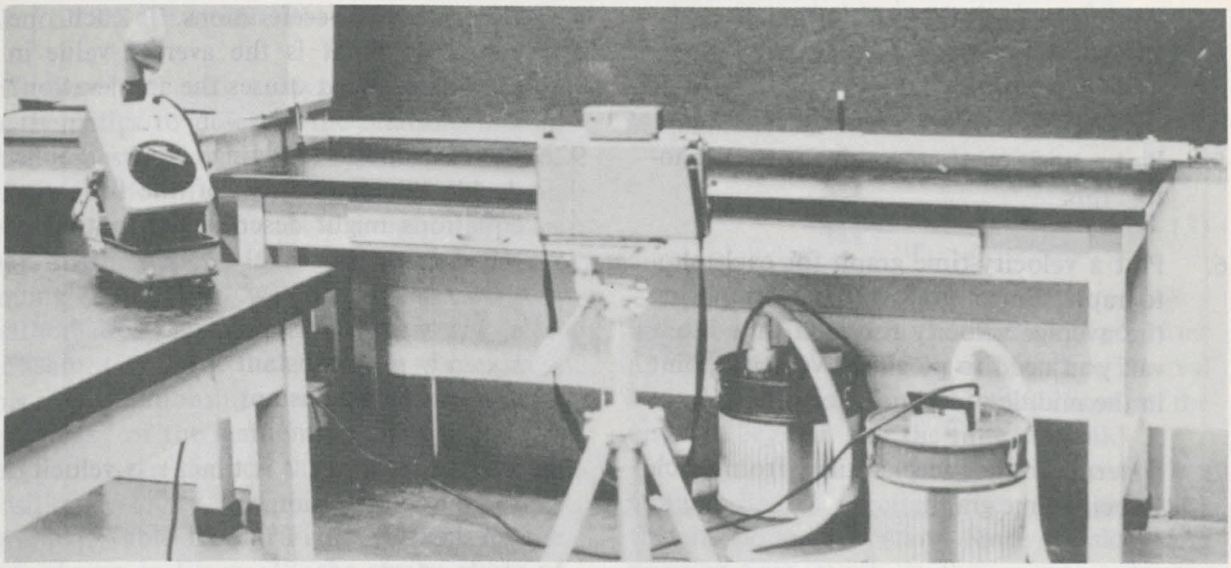


Figure 37.

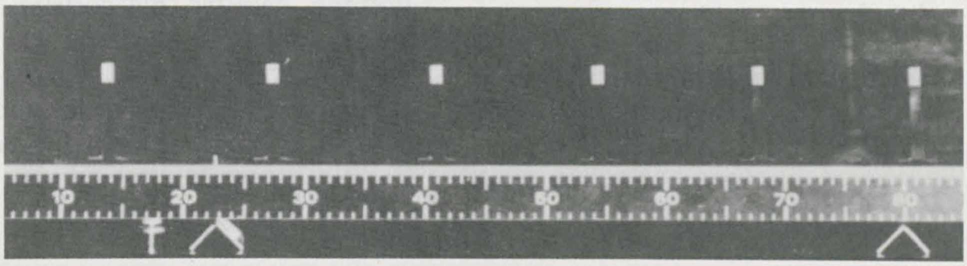


Figure 38.

Table II. Motion of Glider on Level Air Track

Flash #	Elapsed Time (s)	Position x(cm)	Change in Position $\Delta x = x_2 - x_1$	Average Velocity $v_{av} = \Delta x/\Delta t$
1				
2				
3				
4				
5				
6				

you have measured, fill in each table. The calculations are similar to those done in Section A.

5. Plot a position-time graph for each photograph.

6. Plot a velocity-time graph for each photograph. Since you will be computing the average velocity for each time interval, you need to plot each velocity point in the middle of a time interval.

7. Determine the acceleration from each velocity-time graph.

8. Describe the accelerations. Is each one constant? What is the average value in each case? What causes the acceleration?

9. By carefully studying your graphs, decide which, if any, of the following equations might describe the motion of the glider and/or ball.

   a. $x = vt + \text{constant}$
   b. $x = at + \text{constant}$
   c. $v = at + \text{constant}$

   x is the position, t is time, v is velocity, and a is acceleration.

ANALYSIS OF UNIFORM MOTION

Now we will analyze the data using mathematics to describe the relations among the various quantities. The idea that position, velocity, and time are all interrelated is an example of an important concept in physics and mathematics. If a relationship exists among quantities, we often can write a mathematical equation expressing the relationship. Graphs of the quantities also express this same relationship; they can be regarded as "pictures" of the mathematical equation. Our problem is to deduce the equations relating position, velocity, and time from the pictures (graphs). Table III and Figure 39 contain the data shown in Figure 38, the strobe photo of uniform (constant velocity) motion.

First consider the position-time graph (Figure 39A). The data points lie on a straight line. This straight line is a "picture" of the mathematical relationship between position and time for the glider moving along a level air track. We already have the "key" we need to write the equivalent equation. The average velocity has been defined as

$$v_{av} = \frac{\text{change in position}}{\text{time interval}} = \frac{\Delta x}{\Delta t} \qquad (2)$$

Mathematically we write this as

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} \qquad (3)$$

where x_2 and x_1 are the measured positions at the end and beginning of the time interval and t_2 and t_1 are the clock readings at the end and beginning of the time interval.

Saying that the position-time graph is a straight line is precisely the same as saying that its slope is constant. Since the slope of a position-time graph is the average velocity, the average velocity is also constant. If v_{av} is not changing, then we can write

$$v_{av} = v = \text{constant} = \frac{x_2 - x_1}{t_2 - t_1} \qquad (4)$$

where v is the velocity at *any* time. Writing this in a different way, we have

Table III.

FLASH #	ELAPSED TIME (s)	POSITION (cm)	ΔX (cm)	V_{av} (cm/s)
1	0	14		
			13.6	68
2	0.2	27.6		
			13.4	67
3	0.4	41		
			13.5	67.5
4	0.6	54.5		
			13.5	67.5
5	0.8	68		
			13.0	65
6	1.0	81		

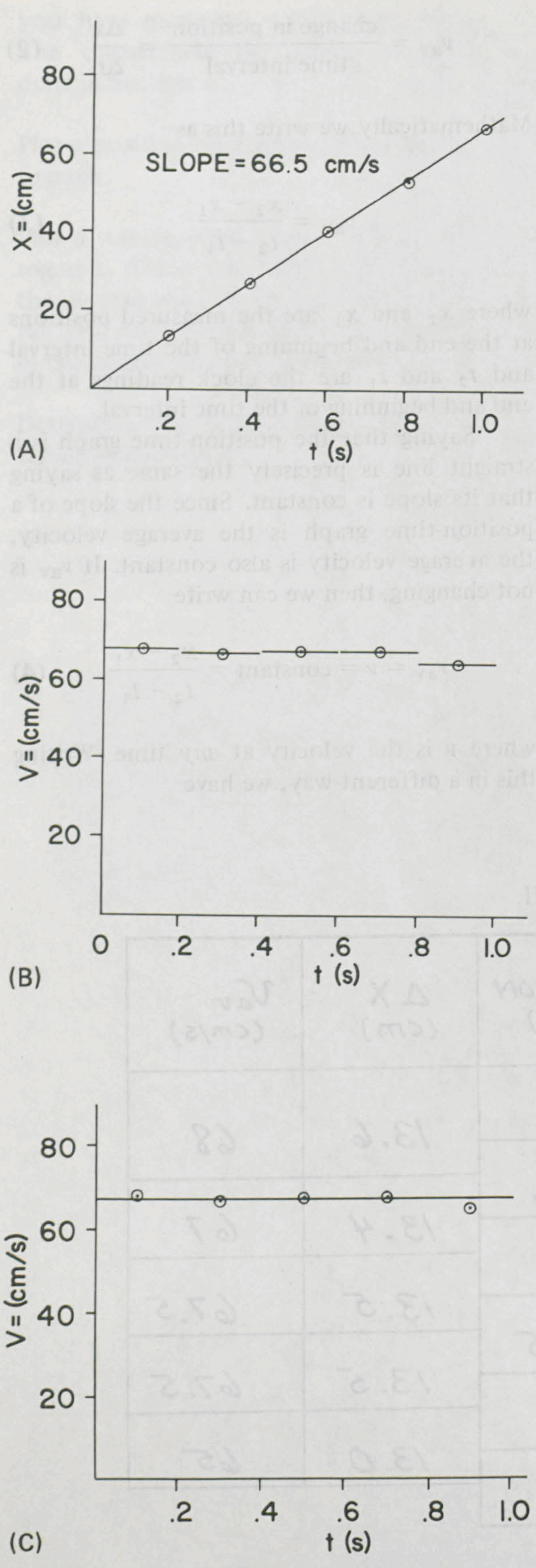


Figure 39.

$$x_2 - x_1 = v \cdot (t_2 - t_1) \qquad (5)$$

This merely says that the change in position (distance moved) is equal to the constant velocity times the corresponding time interval. That is,

$$\text{Distance} = \text{velocity} \times \text{time}$$

We can relate this to our graph of position and time by noting that the position at $t = 0$ is some value which we label x_0. If we allow the time interval in Equation (5) to begin at $t = 0$, then $t_1 = 0$ and $x_1 = x_0$. Equation (5) then becomes

$$x_2 - x_0 = v \cdot (t_2 - 0)$$

or

$$x_2 = x_0 + vt_2$$

But x_2 and t_2 can refer to any point on the straight line. So we can omit the subscripts with the understanding that for any time t, the position is given by

$$x = x_0 + vt \qquad (6)$$

Equation (6) represents the graph in Figure 39A if v is the computed slope (0.66 m/s) and if $x_0 = 0$. Equation (6) is a completely general equation valid for *any* constant velocity v.

Problem 13. Make use of Equation (6) with $v = 10$ m/s and $x_0 = 5$ m to construct a graph of x versus t. How does this graph compare to Figure 39A?

Equation (6) is also general in another sense. If the graph of *any* two quantities X and Y is a straight line, the mathematical relation between X and Y has the same form as that of Equation (6). If X is the horizontal coordinate and Y is the vertical coordinate, then

$$Y = Y_0 + kX \qquad (7)$$

where k is the constant slope of the straight line and Y_0 is the value of Y at $X = 0$. Any relation whose graph is a straight line can be represented by an equation like Equation (7), and is said to be a *linear relation.* If the straight line goes through the *origin* of the graph ($X = 0$, $Y = 0$), so that $Y_0 = 0$, the two quantities are *proportional* to each other.

What about the acceleration? Since the velocity does not change, the glider is not being accelerated. Its acceleration is zero. This is confirmed by the graph of velocity and time in Figure 39B. The slope of the graph is zero, as it should be, since the slope of the velocity-time graph *is* the acceleration.

The unevenness of the velocity-time graph of Figure 39B results from experimental error. One way to get around this is to get the velocity from the position-time graph of Figure 39A. Drawing the straight line that most nearly fits the data points is a way of averaging the data, and the slope of that line gives the constant velocity. Doing this results in the graph of Figure 39C, a smoothed-out velocity-time graph.

ANALYSIS OF UNIFORMLY ACCELERATED MOTION

The following paragraphs apply to both the motion of the glider moving on a tilted air track (as in Figure 40) and the motion of a falling ball (as in Figure 41). The flash rate was 600 fpm in both photographs. Although the analysis is the same for both types of motion, for convenience we will discuss only the falling ball. Table IV and Figure 42 summarize the data contained in the glider photograph (Figure 40), while Table V and Figure 43 summarize the data contained in the photograph of the falling ball (Figure 41).

Since straight-line graphs are easier to analyze than curved graphs, and since we have just completed an analysis of a straight-line graph, we begin by considering the velocity-time graph (Figure 43A). As for uniform motion, an analysis of the position-time graph could be done. However, since the graph is a straight line, we can use Equation (7), which is a general formula for any straight line. If we let $Y = v$, $X = t$, and $Y_0 = v_0$, Equation (7) becomes

$$v = v_0 + kt$$

where v is the velocity at time t, v_0 is the velocity at time $t = 0$, and k is the slope of the v-t graph. But the slope of a velocity-time graph is just the acceleration a, so we have

$$v = v_0 + at \qquad (8)$$

The fact that the velocity-time graph is a straight line (constant slope) indicates that the acceleration of the ball is constant. Equation (8) is true only for constant acceleration. It says that the velocity v after t seconds is equal to the initial velocity v_0 plus the change in velocity due to the acceleration (at). You should realize that in general an acceleration may act to decrease the velocity (negative acceleration) as well as to increase it (positive acceleration).

You can find the value of the acceleration of the ball by computing the slope of your velocity-time graph. From Figure 43A, the acceleration is

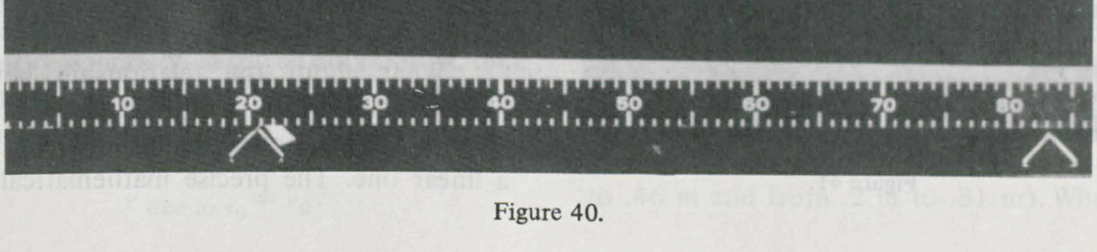


Figure 40.

$$a = \frac{5.4 - .8 \text{ m/s}}{.45 \text{ s}} \cong 10 \text{ m/s}^2$$

The downward acceleration of the falling ball is called the *acceleration due to gravity g.* In the United States, g varies from about 9.82 m/s^2 in Alaska to about 9.79 m/s^2 in Florida. For ordinary calculations on the surface of the earth, g is regarded as a constant equal to 9.8 m/s^2.

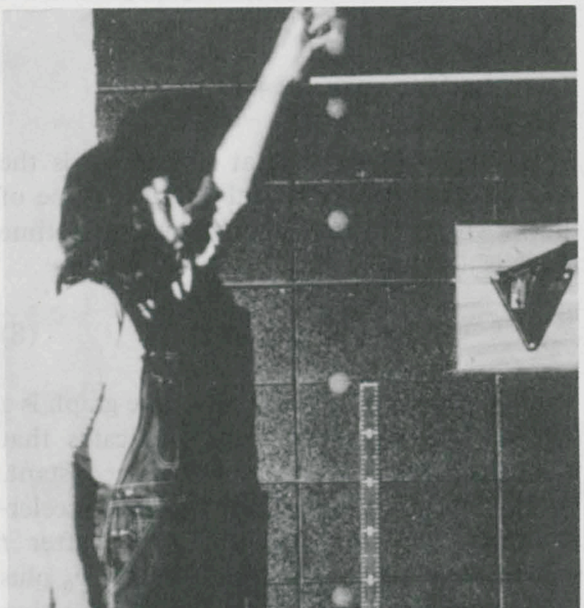


Figure 41.

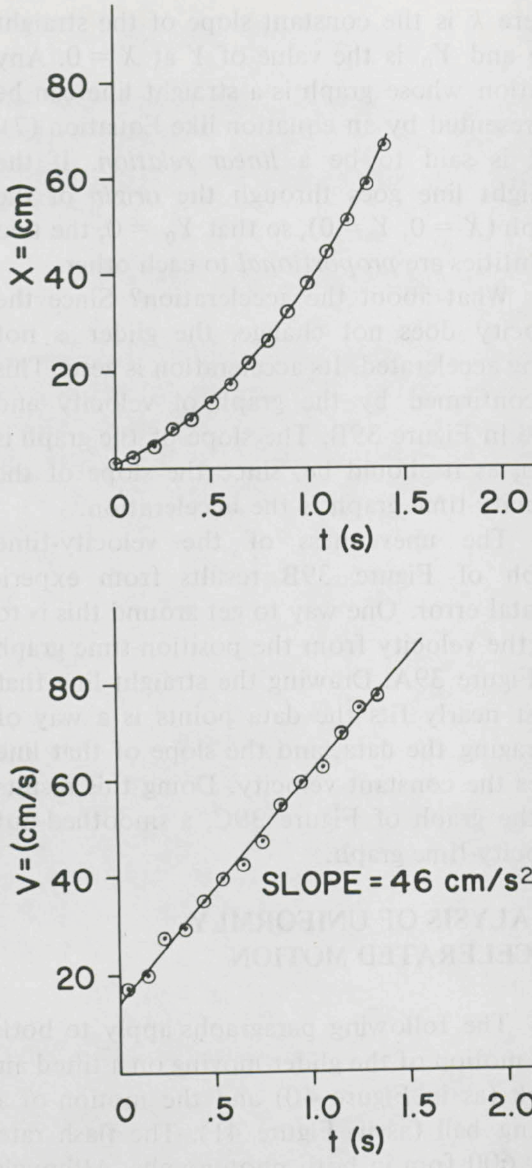


Figure 42.

If you photographed the glider on a tilted air track, you probably found a constant acceleration which was much less than g. The reason for this will be explained briefly in Section C of the module.

Both the glider and the ball are examples of motion involving constant acceleration. Such motion is called *uniformly accelerated motion.*

What about the relationship between position and time? The position-time graph (Figure 43B) indicates that the relation is not a linear one. The precise mathematical form

Table IV.

FLASH #	ELAPSED TIME (s)	POSITION (cm)	Δx (cm)	$\bar{v}_{ave}$ (cm/s)
1	0	0.5	1.8	18
2	0.1	2.3	2.0	20
3	0.2	4.3	2.7	27
4	0.3	7	3.0	30
5	0.4	10	3.5	35
6	0.5	13.5	4.0	40
7	0.6	17.5	4.3	43
8	0.7	21.8	4.7	47
9	0.8	26.5	5.5	55
10	0.9	32	6.0	60
11	1.0	38	6.3	63
12	1.1	44.3	7.0	70
13	1.2	51.3	7.5	75
14	1.3	58.8	7.7	77
15	1.4	66.5		

can be determined by an examination of the graphs and the data.

You know that the equation for Figure 43A is $v = v_0 + at$, and you can see from the graph that the initial velocity v_0 of the ball at time $t = 0$ was not zero. At any time t, the velocity can be thought of as having two parts, the original velocity v_0 that it had at $t = 0$ when we started to measure time, plus the added velocity at that it has gained due to acceleration since the clock started. If we want to find the total distance that the accelerating ball travels in any time t, it is easiest to consider these two parts of the velocity separately. You already know that the distance that the ball will travel as a result of the (constant) initial velocity part will be:

$$x_{\text{due to } v_0} = v_0 t$$

In order to analyze the part of the distance covered which results from the acceleration term (at), we will have to look at it alone, with the v_0 term removed. We can do this by subtracting the $v_0 \cdot t$ term from each position, as shown graphically in Figure 44. Table VI shows how to adjust the data from Table V by subtracting $v_0 t$ from the corresponding position x for each value of time. Figure 45 is a graph of the adjusted position $(x - v_0 t)$ versus time t. It is a graph of the acceleration contribution alone, because the constant velocity contribution has been subtracted from each term.

Notice that in Figure 45, when the time doubles (from $t = .15$ s to $t = .3$ s or from $t = .2$ s to $t = .4$ s), the distance $(x - v_0 t)$ goes up by a factor of about four (from .12 m to .46 m and from .2 m to .81 m). When the

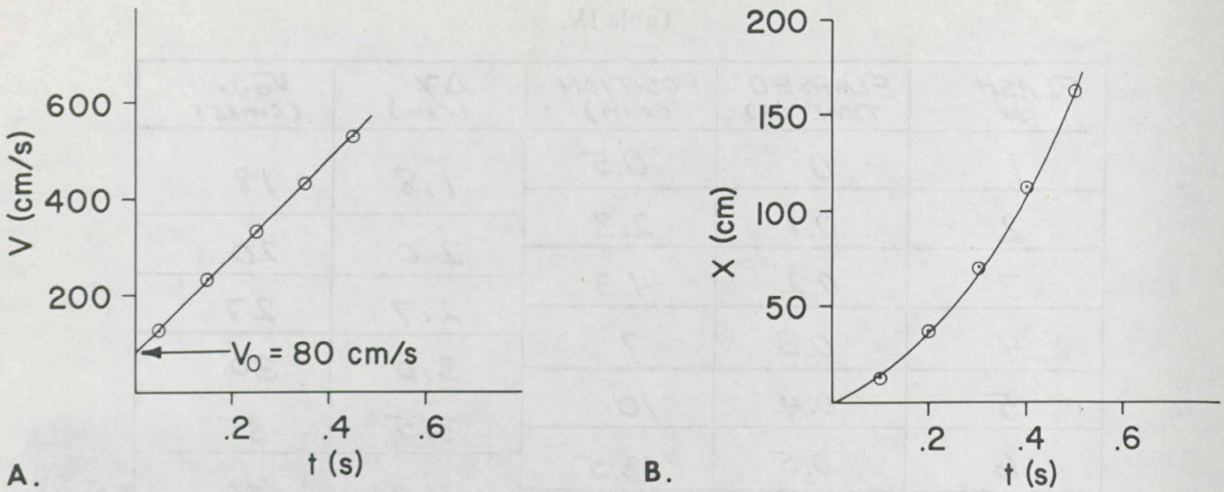


Figure 43.

Table V.

FLASH #	ELAPSED TIME (s)	POSITION (cm)	Δ X (cm)	V_{ave} (cm/s)
1	0	0	12.5	125
2	0.1	12.5	23.8	238
3	0.2	36.3	33.7	337
4	0.3	70	43.8	438
5	0.4	113.8	53.7	537
6	0.5	167.5		

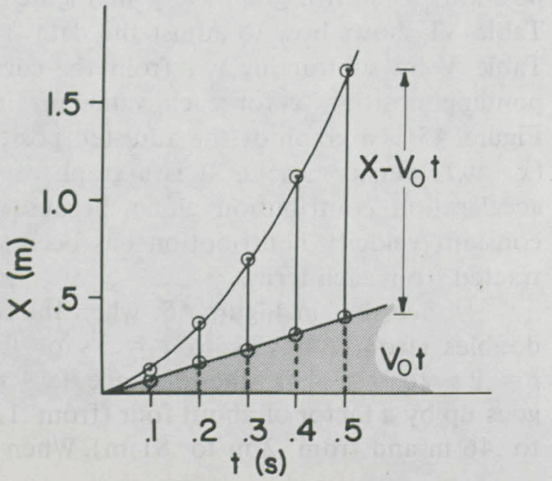


Figure 44.

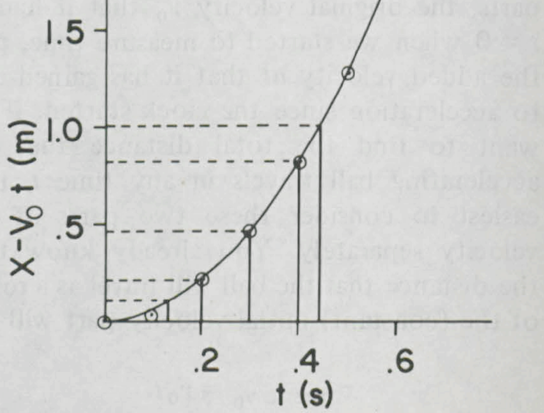


Figure 45.

Table VI.

FLASH #	POSITION (m)	TIME (s)	$v_0 t$ (m)	$x - v_0 t$ (m)	t^2 (s^2)
1	0	0	0.0	0	0
2	0.125	0.1	0.08	0.045	0.01
3	0.363	0.2	0.16	0.203	0.04
4	0.7	0.3	0.24	0.46	0.09
5	1.138	0.4	0.32	0.818	0.16
6	1.675	0.5	0.4	1.275	0.25

time increases by a factor of three (from $t = .15$ s to $t = .45$ s) the position $(x - v_0 t)$ increases by a factor of nine (from 0.12 m to 1.1 m). This suggests that the change in position $(x - v_0 t)$ is proportional to the square of time (t^2). That is,

$$x - v_0 t \propto t^2$$

How can we verify this? A simple way is to graph $x - v_0 t$ versus t^2. If the graph is a straight line passing through the origin, then the two quantities are proportional and we know how to write the equation relating them. Figure 46 is a graph of $x - v_0 t$ on the vertical axis and t^2 on the horizontal axis. It *is* a straight line through the origin. We know that the equation of such a graph has the form

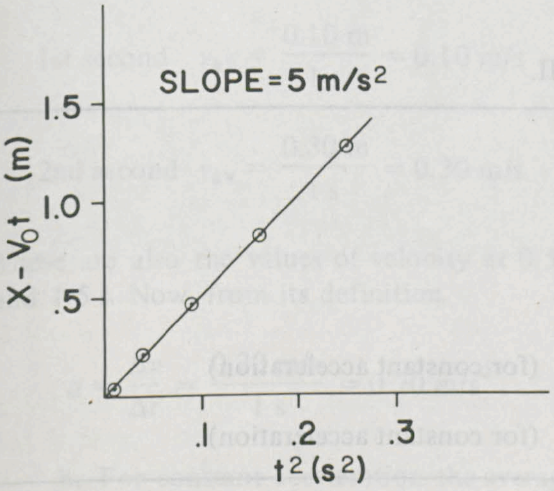


Figure 46.

$$Y = Y_0 + kX$$

In plotting Figure 46, we have used $x - v_0 t$ as the vertical coordinate Y, t^2 as the horizontal coordinate X, and Y_0 is zero. Now the relation between position and time can be written as

$$x - v_0 t = 0 + kt^2 \quad (9)$$

The value of k is the slope of the straight line. For Figure 46 this value is 5 m/s^2. But the acceleration of gravity is 9.8 m/s^2. Since the change in position is due to gravitational acceleration, it makes sense that k is related to g. By comparing the numbers, we see that $k \approx \frac{1}{2} g$. More accurate experiments show that

$$x - v_0 t = \tfrac{1}{2} g t^2$$

or

$$x = v_0 t + \tfrac{1}{2} g t^2 \quad (10)$$

Equation (10) says that the position of a moving object (in this case, a freely falling ball) after t seconds is given by the sum of two terms. The first of these is the distance the object would travel at constant velocity v_0 in t seconds, and the second is the additional distance it travels in t seconds as a result of the fact that its velocity is changing due to a constant acceleration. For *any* constant acceleration a, we can write

$$x = v_0 t + \tfrac{1}{2} a t^2 \qquad (11)$$

In finding Equations (10) and (11), we took the position x at $t = 0$ to be zero. Similarly, in Experiment B-1, you placed the ruler on an image of the falling ball and called the position of that image $x = 0$. But we might want to measure x from some other origin. For instance it is likely that the ball had already fallen some distance x_0 before the strobe flashed to record the image you picked for $x = 0$. If we want to measure position from the point at which the ball is released, we can add the position (x_0) of the ball at $t = 0$ to Equation (10) or Equation (11). Thus

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad (12)$$

The value of x_0 always depends on where the origin of the graph is located. It is always possible, and often preferable, to choose the origin so that $x_0 = 0$. But it is not necessary to do so.

Table VII summarizes the equations of motion.

The following examples will illustrate how these equations can be used.

Example 9. A car accelerates uniformly from 22 m/s to 44 m/s in 11 s.

a. What is the acceleration of the car?

b. What is its average velocity during that time?

c. How far does it travel during the 11 s?

Solution. a. From the definition of acceleration

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{44 \text{ m/s} - 22 \text{ m/s}}{11 \text{ s}} = 2 \text{ m/s}^2$$

b. For constant acceleration, the average velocity during a time interval is just the velocity in the middle of that interval. One way to find this is to average the initial and final values of velocity. That is

$$v_{av} = \frac{v_0 + v_f}{2}$$

$$= \frac{44 + 22 \text{ m/s}}{2} = 33 \text{ m/s}$$

c. One easy way to find the distance traveled during the 11 s is to use the definition of average velocity and the value of v_{av} just computed.

$$v_{av} = \frac{\Delta x}{\Delta t} = 33 \text{ m/s}$$

$$\Delta x = v_{av} \cdot \Delta t = 33 \text{ m/s} \times 11 \text{ s} = 363 \text{ m}$$

There are many other ways to solve this same problem. For example, after calculating the acceleration, the distance traveled can be found using Equation (12)

Table VII.

$v_{av} = \dfrac{\Delta x}{\Delta t}$	
$a_{av} = \dfrac{\Delta v}{\Delta t}$	
$v = v_0 + at$	(for constant acceleration)
$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$	(for constant acceleration)

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$= 0 + 22 \text{ m/s} \cdot 11 \text{ s} + \frac{1}{2} (2 \text{ m/s}^2)(11 \text{ s})^2$$

$$= 0 + 242 \text{ m} + 121 \text{ m} = 363 \text{ m}$$

Then the average velocity can be calculated using the definition, Equation (3).

Example 10. Measurements are made from a strobe photo of a glider on an air track. They provide the following results for the positions of the first three images.

1st image $x = 0.0$ m

2nd image $x = 0.10$ m

3rd image $x = 0.40$ m

The strobe rate is one flash per second. It is known that the acceleration is constant. Determine:

a. The acceleration

b. The velocity at $t = 0$

c. The distance between the 4th and 7th images

Solution. a. The average velocity during the first second and during the second second can be calculated from the definition of average velocity:

$$\text{1st second} \quad v_{av} = \frac{0.10 \text{ m}}{1 \text{ s}} = 0.10 \text{ m/s}$$

$$\text{2nd second} \quad v_{av} = \frac{0.30 \text{ m}}{1 \text{ s}} = 0.30 \text{ m/s}$$

These are also the values of velocity at 0.5 s and 1.5 s. Now, from its definition

$$a = \frac{\Delta v}{\Delta t} = \frac{0.20 \text{ m/s}}{1 \text{ s}} = 0.20 \text{ m/s}^2$$

b. For constant acceleration the average velocity during a time interval is equal to the value the velocity reaches at the midpoint of the time interval. Applying Equation (8) we get

$$v = v_0 + at$$

This can be rearranged as

$$v_0 = v - at$$

During the first second the average velocity is 0.1 m/s. It had an instantaneous velocity equal to the average velocity at the middle of the time interval. In other words, when $t = 0.5$ s, $v = 0.10$ m/s. Using this and the known acceleration, $a = 0.2 \text{ m/s}^2$, we get

$$v_0 = 0.10 \text{ m/s} - (0.2 \text{ m/s}^2)(0.5 \text{ s})$$

$$= 0.10 \text{ m/s} - 0.10 \text{ m/s} = 0.0 \text{ m/s}$$

In this particular case, then, the initial velocity (the velocity at $t = 0$) is zero.

c. There are several different ways to solve the last part of the problem. The most straightforward is to use Equation (12) for the two times. (Note that the 4th image occurs at $t = 3$ s and the 7th at $t = 6$ s, because the first image appears at $t = 0$.) This gives us:

$$x_4 = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$= 0 + 0 + \frac{1}{2} (0.2 \text{ m/s})(35)^2$$

$$= 0.9 \text{ m}$$

$$x_7 = 0 + 0 + \frac{1}{2} (0.2 \text{ m/s})(6 \text{ s})^2$$

$$= 3.6 \text{ m}$$

$$x_7 - x_4 = 3.6 \text{ m} - 0.9 \text{ m} = 2.7 \text{ m}$$

Problem 14. A driver travels 160 mi in 3 h, stops 1 h for lunch, and drives 170 more mi in 3.5 h. Find:

a. The average speed for the first 160 mi

b. The average speed for the 170-mi portion

c. The average speed for the whole trip

Problem 15. The "fastball" pitched by a major league player travels about 7 ft in a nearly straight line as it is being thrown (twice arm's length plus shoulder motion). The time it takes to throw it is about 0.2 s.

a. Find the average speed of the baseball as it is being thrown.

b. Assume that the speed of the baseball changes uniformly and find the maximum speed of the ball.

c. Find the acceleration.

Problem 16. The C-5A military transport requires 1.25 mi of runway to reach its take-off speed of 220 ft/s. Assuming the acceleration to be constant, calculate

a. The average speed during take-off

b. The take-off time

c. The acceleration of the aircraft

Problem 17. The 7.62 mm NATO rifle fires a bullet at a velocity of 900 m/s from a rifle barrel 0.75 m long.

a. Find the time the bullet spends in the barrel.

b. Find the acceleration of the bullet.

Problem 18. In an automotive braking test an accelerometer (a device used to measure accelerations) measured 7.8 m/s^2. How much distance was required to stop the car? The initial speed was 100 km/h (1 km/h = 0.28 m/s).

Motion in Two Dimensions

The motions which we have considered so far in this module have been *linear motions*, motions which occur along a straight line. However, many motions are not confined to straight lines. One very common case is motion in a plane, two-dimensional motion. We see one example of this motion when we throw a ball, and you will look at such motion in Experiment B-2.

EXPERIMENT B-2. Motion in a Plane

In this experiment, the stroboscope will again be used as a tool for the analysis of the motion of a ball.

Procedure

1. Set up the strobe light and camera in front of a dark background and use a reference, such as a meter stick, as you did in Experiment B-1. Darken the room with the strobe light flashing and open the shutter of the camera as a white ball is thrown in front of the background. Someone in the class should take at least one picture with the ball as it is projected at various initial angles to the horizontal. Figure 47 shows an example of such a photograph. A flash rate of 1200 fpm gives you a useable time interval and a good number of images. Be sure to record the flash rate.

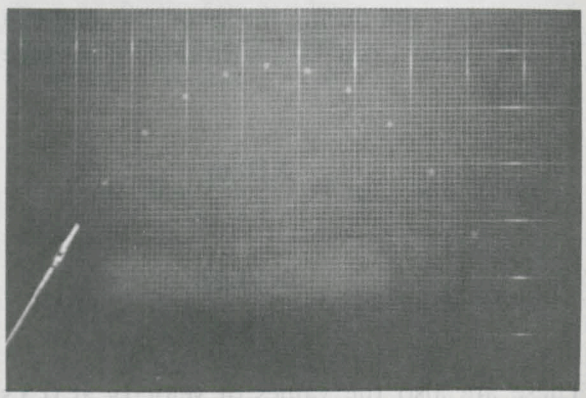


Figure 47.

2. At first glance, the path of the ball looks different from the other motions we have studied so far. The photograph is essentially a graph of the vertical distance traveled versus the horizontal dis-

tance traveled. However, if you examine the horizontal motion and the vertical motion *separately,* you will find that each of these parts of the motion is similar to a motion already studied. The horizontal and vertical positions of each image of the ball can be measured from the scale in the photograph. Measure successive positions of the ball for as many flashes as you can. One convenient way to do this is to punch a hole with a pin through each image of the ball. Then, using an overhead projector, project the pinholes on a chalkboard and mark each position on the board. If you are clever about this, you can make the path on the chalkboard come out to exactly the same size as the real path of the ball. Each position should be measured from the position of the first image.

3. From the flash rate of the strobe, calculate the time Δt between flashes.

4. Construct a table for both the horizontal distance x and the vertical distance y at each time.

5. Plot a position-time graph for both the vertical and the horizontal motions. Take distances to the right of the first image to be positive ($+x$) and distances above the first to be positive ($+y$). What is the nature of each motion?

6. Plot a velocity-time graph for both the vertical and horizontal motions. If either of the position-time graphs is a straight line, you can get the constant velocity of that motion by just measuring the slope of the line.

7. Comparing these graphs with those in Section A, how would you describe the vertical motion?

Table VIII summarizes the motion in the horizontal (x) direction and in the vertical (y) direction for the photograph of Figure 47.

Table VIII.

FLASH #	ELAPSED TIME (s)	X-POSITION (cm)	Δx (cm)	$v_{x\,av}$ (cm/s)	Y-POSITION (cm)	Δy (cm)	$v_{y\,av}$ (cm/s)
1	0	0			0		
			7	140		9	180
2	0.05	7			9		
			7	140		6.5	130
3	0.10	14			15.5		
			7.5	150		4	80
4	0.15	21.5			19.5		
			7.5	150		1.5	30
5	0.20	29			21		
			7	140		−1.2	−24
6	0.25	36			19.8		
			7.5	150		−3.5	−70
7	0.30	43.5			16.3		
			7.5	150		−6	−120
8	0.35	51			10.3		
			7	140		−8.5	−170
9	0.40	58			1.8		
			7.5	150		−11.3	−226
10	0.45	65.5			−9.5		

From both the original photograph and the graph of Figure 48A, we see that the ball moves equal horizontal distances between strobe flashes. This implies that the horizontal velocity is constant, and Figure 48B confirms that it is. Thus, the horizontal acceleration is zero. It is as if the ball were not falling at all, but moving at constant speed on a level surface.

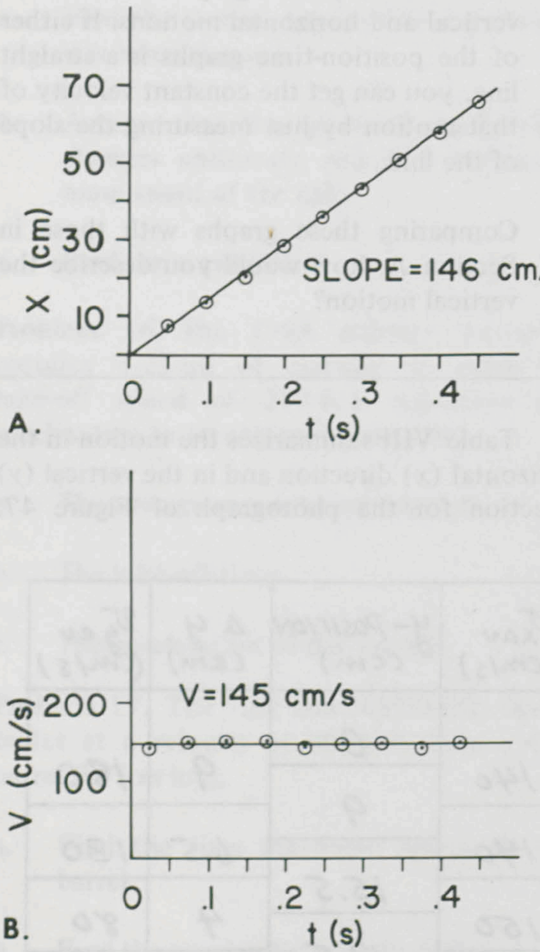


Figure 48.

Although it may not seem so at first glance, the vertical position-time graph is almost identical to that of the falling ball previously studied. The vertical velocity-time graph is a straight line, which means that the vertical acceleration is constant. You might suspect that this vertical acceleration is related to the acceleration due to gravity. After all, if the ball were simply dropped instead of thrown, the downward acceleration would be that of gravity. Measurement of the slope of Figure 49 yields a value for the acceleration of about -1000 cm/s^2. This is very close to the accepted value for g (980 cm/s^2). The minus sign means that the acceleration acts in a direction opposite to that chosen as the positive y-direction (i.e., downward). In the analysis of Experiment B-1, the downward direction was chosen to be positive, so the measured value for g turned out to be positive. The vertical part of the motion of the ball differs from the vertical motion of the ball in Experiment B-1 only because the positive y-direction was chosen differently, and because this experiment showed the ball rising to its highest point, then falling from it.

Since the acceleration is constant, the equations of motion in Table VII apply and we can write, for vertical position y,

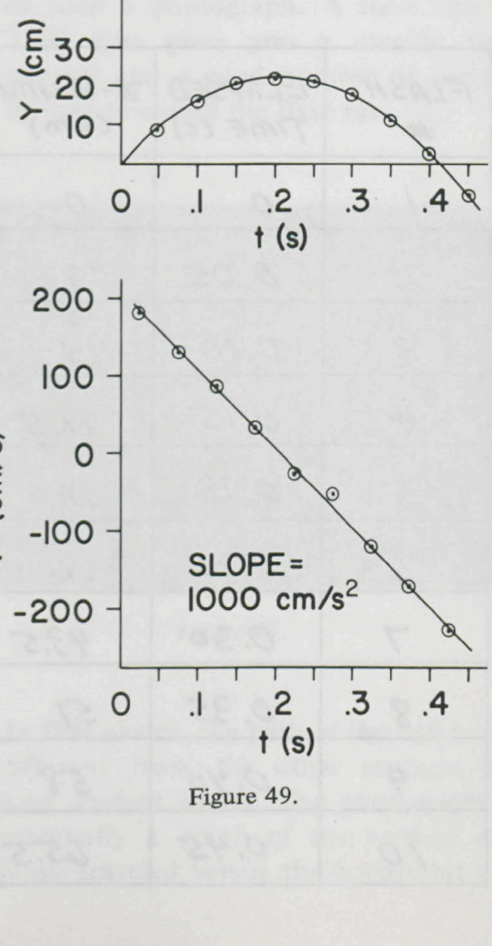


Figure 49.

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

Note that the downward direction of the acceleration is included in the equation by virtue of the negative sign. If the ball had been projected with initial vertical velocity v_{0y} directed downward, then the $v_{0y}t$ term would also be negative.

We can summarize the horizontal and vertical motion in equation form as in Table IX.

Table IX.

Horizontal	Vertical
$a_x = 0$	$a_y = -g$
$v_x = v_{0x} = \text{constant}$	$v_y = v_{0y} - gt$
$x = v_{0x}t$	$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$

Although it may seem strange that we can study the horizontal and vertical parts of the motion independently, it is always easier to separate a problem into its component parts and study each part separately. Figure 50 shows how the vertical and horizontal parts of the motion are related for the downward portion of the ball's trajectory. The relationship is simply that both motions occur at the same time.

Figure 51 shows an experiment which demonstrates even more clearly that the vertical motion of a projectile is the same as that of a freely falling ball. The strobe rate is 1200 fpm for this photo. The ball which falls straight down is suspended from an electromagnet which is turned off by the other ball as it leaves the gun. Thus, the hanging ball is released just as the other ball leaves the gun, and both balls start to fall at the same time. One ball falls directly downward, and the other has a horizontal motion as well. You can compare the vertical motion of the two balls. What has happened between the 4th and 5th images of the projectile? What would happen if the projectile were shot slower? Faster?

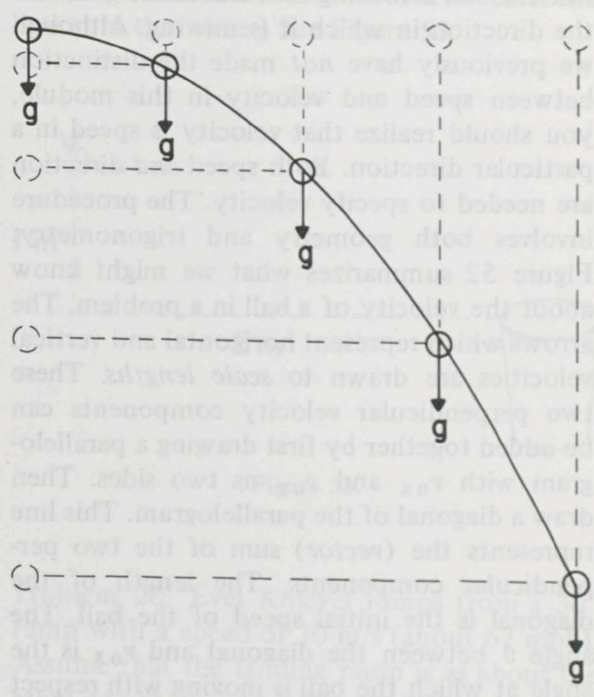


Figure 50.

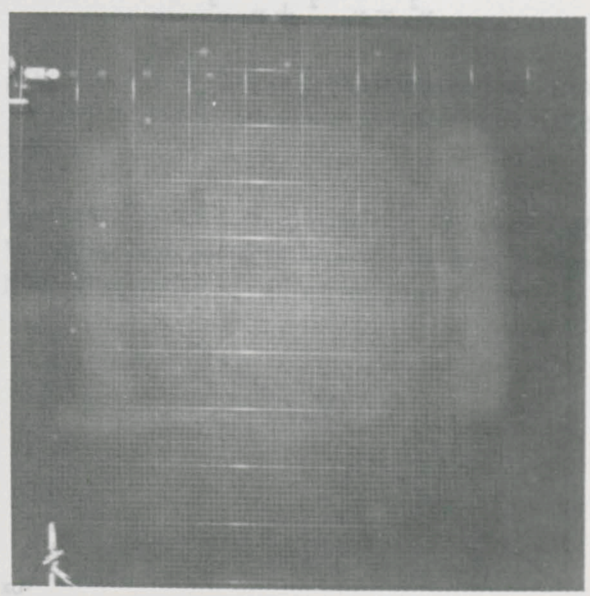


Figure 51.

INITIAL VELOCITY OF A PROJECTILE (OPTIONAL)

You have measured values of horizontal and vertical velocities from the stroboscopic data. We now can extend the analysis to write a single value of the velocity which tells how fast the ball is moving as it leaves the gun, and the direction in which it is moving. Although we previously have *not* made the distinction between speed and velocity in this module, you should realize that velocity is speed in a particular direction. Both speed *and* direction are needed to specify velocity. The procedure involves both geometry and trigonometry. Figure 52 summarizes what we might know about the velocity of a ball in a problem. The arrows which represent horizontal and vertical velocities are drawn to *scale lengths.* These two perpendicular velocity components can be added together by first drawing a parallelogram with v_{0x} and v_{0y} as two sides. Then draw a diagonal of the parallelogram. This line represents the (*vector*) sum of the two perpendicular components. The length of the diagonal is the initial speed of the ball. The angle θ between the diagonal and v_{0x} is the angle at which the ball is moving with respect to the horizontal. From the Pythagorean Theorem the length of the diagonal is

$$v^2 = v_{0x}^{\ 2} + v_{0y}^{\ 2}$$

$$v = \sqrt{v_{0x}^{\ 2} + v_{0y}^{\ 2}} \tag{13}$$

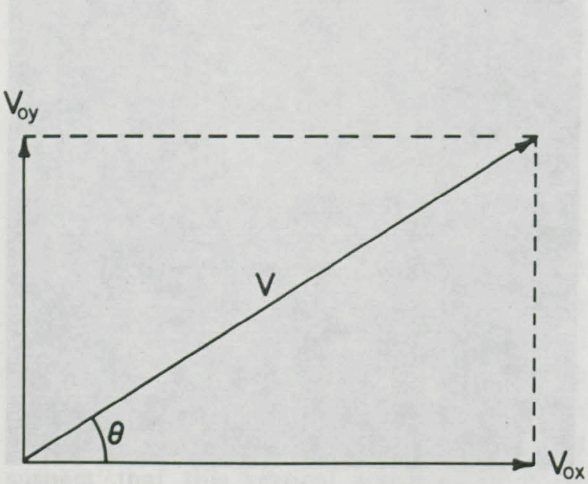


Figure 52.

The tangent of the angle θ is computed by trigonometry to be

$$\tan\theta = \frac{v_{0y}}{v_{0x}}$$

Therefore

$$\theta = \text{arc tan}\,\frac{v_{0y}}{v_{0x}} \tag{14}$$

In many cases, the inverse of the problem arises; we know the initial speed of the ball and the angle that its velocity makes with the horizontal, and we want to find the vertical and horizontal components of the velocity. Frequently, this case arises when we know the initial speed and direction of a ball and we have to split the initial velocity into components to be used for the initial speeds in the horizontal and vertical equations. To do this, we can just reverse the approach described above. The procedure is shown in Example 11.

Example 11. At the kick-off from the 40-yard line, a football is kicked so that it initially has a speed of 20 yd/s directed at an angle of 30° above the horizontal. The kick is straight down the field. About where will it land?

Solution. The units "yards per second" are a little strange, but appropriate to this problem. Perhaps in the near future our football fields will be measured in meters.

The part of the motion of the football in which we are interested starts when it is kicked and ends when it hits the ground. The time between these events is the length of time that the ball is in the air. If we can calculate this time, we can compute the distance traveled down the field.

The initial velocity of the ball as shown in Figure 52 can be broken into horizontal and vertical components. The time that the ball is in the air can be found by considering how long it takes a ball with initial vertical velocity $v_{0y} = v_0 \sin 30° = v_0\,(½)$ to go up

and come back down. The vertical position is given by

$$y = v_{0y} \cdot t - \tfrac{1}{2} gt^2$$

In our strange units, $g = 32\ \text{ft/s}^2 \approx 10.7\ \text{yd/s}^2$. When the ball hits the ground, $y = 0$ and we have

$$0 = \tfrac{1}{2}\, v_0 \cdot t - \tfrac{1}{2}\,(10.7\ \text{yd/s}^2)\, t^2$$

This is a quadratic equation, which has two possible solutions. One solution is $t = 0$ s. We already know that $y = 0$ at $t = 0$ s, because $t = 0$ s is the time at which the ball is kicked. The other solution for t can be found by dividing both sides of the equation by t and rearranging to get:

$$\frac{10.7}{2}\, t = \frac{1}{2}\, v_0$$

$$t = \frac{20\ \text{yd/s}}{10.7\ \text{yd/s}^2} \cong 1.9\ \text{s}$$

The ball is moving horizontally with constant horizontal velocity $v_{0y} = v_0 \cos 30^\circ = 0.87\, v_0$ during this same time. Thus, the distance traveled is

$$x = v_{0x} \cdot t = 0.87\, v_0 t$$

$$= 0.87\,(20\ \text{yd/s})(1.9\ \text{s})$$

$$\cong 33\ \text{yd}$$

This hits just inside the 25-yard line, and we have ignored air resistance–not a very good kick.

Problem 19. A student wants to throw a wadded paper ball into a waste basket which is two meters away from him and one meter below his hand as he throws. If he throws the ball horizontally, how fast must it be thrown to hit the waste basket? (Hint: first calculate how long the paper ball will be in the air and then how fast it must be thrown to reach the basket in that time.) See Figure 53.

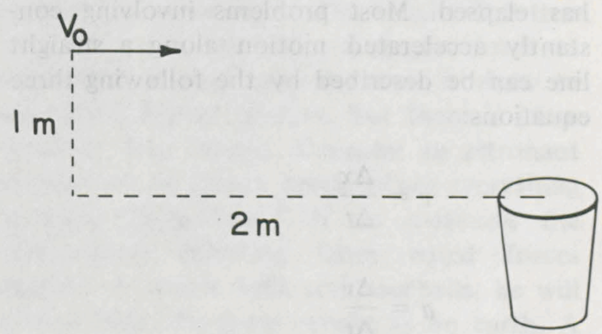


Figure 53.

Problem 20. Evel Knievel jumps from a 30° ramp with a speed of 30 m/s (about 67 mph). Assume that the landing ramp is at about the same height as the one from which he jumped and calculate the length of the jump he can make.

Problem 21. A ball is thrown straight up into the air. Describe precisely the motion of the ball.

Problem 22. Sketch graphs for the position, velocity, and acceleration of the ball of the preceding problem–all versus time.

SUMMARY

In your study of kinematics, you have seen how certain kinds of motion involving constant acceleration can be described. For example, to describe the motion of a ball which has been dropped, an equation was written relating the distance the ball falls to the time required. You learned how to compute the acceleration of the ball and the speed at which it is falling after a given time has elapsed. Most problems involving constantly accelerated motion along a straight line can be described by the following three equations:

$$v = \frac{\Delta x}{\Delta t}$$

$$a = \frac{\Delta v}{\Delta t}$$

$$x = x_0 + v_0 t + ½ at^2$$

where v is the average velocity during a time interval Δt, Δx is the distance moved in that interval, a is the average acceleration during Δt, and Δv is the corresponding change in speed. x is the distance traveled in time t under constant acceleration a. All falling bodies on earth experience the same gravitational acceleration, approximately 9.8 m/s^2.

Each of the first two equations can be "turned around" to allow computation of distance traveled or velocity change during Δt if average velocity and/or average acceleration are known. That is,

$$\Delta x = v\Delta t$$

$$\Delta v = a\Delta t$$

If the accelerations are constant, motion in a single plane can be described in terms of the same three equations. Then, the motion is broken up into perpendicular components.

For projectile motion, the most convenient components are along the horizontal and vertical directions. The vertical motion is the same as that of a freely falling object, while the horizontal motion is uniform motion at constant speed.

SECTION C

Newton's Laws

Just having a description of the motion is not completely satisfactory from a physics viewpoint. For example, you might wonder why a falling ball accelerates and a falling parachute does not. Why does the glider on the level air track move at constant speed? To answer these questions (and many others) we need to study *dynamics,* which considers the cause of motion. This section is a brief introduction to dynamics.

NEWTON'S SECOND LAW

Everyone knows that the harder one hits or throws a ball, the faster the ball moves. Although this is common knowledge, an important physical concept underlies it. To accelerate an object, you must push on it. That is, you must exert a *force* on it. You intuitively know that a force is a push or a pull. A push or a pull is always necessary to change the shape of an object, or to alter the motion of an object. Although it is difficult to describe force more precisely, it turns out that we can easily specify ways to measure force. One such definition of force is in terms of the pull necessary to stretch a given spring a certain distance. Many other "working" definitions of force are possible. Here we will examine the relationship between the force exerted on an object and the resulting acceleration.

Is it sufficient to consider only the force exerted on an object and the resulting acceleration? Does knowledge of the force on an object allow one to predict its acceleration? Clearly not, as the following example shows. If you exert the same force on a car as you exert on a motorcycle, the motorcycle will accelerate much faster than the car. Thus, the car must have some property that makes it harder to accelerate.

One might decide that the acceleration of an object depends not only on the force exerted on it but also on its weight. It is true that, on earth, it takes a greater force to accelerate heavier objects than it does to accelerate lighter objects. But there is more involved than weight. Consider an astronaut somewhere in empty space where everything is truly "weightless." If he measures the acceleration resulting from equal forces applied to tennis balls and baseballs, he will always find the same results as on earth. A greater force is required to give the baseball the same acceleration as the tennis ball. Since both the baseball and the tennis ball have the same weight in the space capsule (zero), some other factor must be involved.

It turns out that this property of an object—the thing that determines how difficult it is to accelerate—is an important basic physical quantity. It is called *mass* (m). Mass is very difficult to define, as is force. Sometimes mass is described as a "measure of the amount of matter in a body." But this is not a very useful definition because it does not tell us *how* to measure the amount of matter in a body. The best way to define mass is to use the relationship between force, acceleration, and mass. That is, an object's mass can be defined as a measure of its resistance to acceleration (also called its *inertia*). Such measurements provide a "working" definition of a quantity. You will investigate this relationship in Experiment C-1.

EXPERIMENT C-1. Force, Mass, and Acceleration

In this experiment, you will analyze strobe photos of motion on the linear air track in order to determine how the acceleration depends on the applied force. The force will be provided by the weight of a mass* hung over a pulley at one end of the air track. To simplify analysis of the experiment, the amount of matter, or mass, which is being accelerated will be kept constant. That is, the sum of the mass of the glider and the hanging mass will always be the same. You can vary the accelerating force (weight on the thread) and measure the resulting acceleration while keeping the total mass constant. Figure 54 illustrates the apparatus.

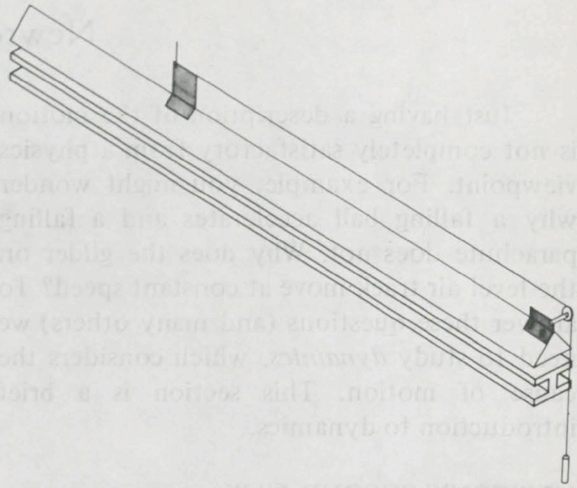


Figure 54.

Procedure

1. Attach several of the available masses to the glider. Be sure that you attach the same number of masses to each side of the glider so that the glider is balanced.

2. Check that the air track is level by making sure that the glider does not move along the track when you release it from rest anywhere on the track. At this point, there is no force applied to the glider (parallel to the track) and no resulting acceleration. If you give the glider a gentle shove along the track, what is its acceleration after you stop pushing?

3. Tie a 20-g mass to a piece of light thread. Tie the other end of the thread to the glider and hang it over the pulley. Record the amount of mass tied to the thread. The weight of this mass is the force which accelerates the system. This force can be measured in units called newtons (N). The force in newtons is the weight in kilograms multiplied by the acceleration of gravity (9.8 m/s^2). You will soon see why this is so.

4. Release the glider from rest and take a stroboscopic picture of it, using the same procedure as in Section B. A white pointer should be fastened to the glider. (See Figure 55.) You may have to experiment to get an appropriate flash rate. About 600 fpm is a good place to start.

5. From measurements on the photograph, plot a velocity-time graph and determine the acceleration from its slope. Refer to Experiment B-1 if you need help here.

6. Remove two masses from the glider, one from each side to maintain balance, and attach them to the lower end of the thread. Again record the mass on the thread.

7. Repeat steps 4 and 5 for the new value of the accelerating weight (force).

8. Repeat steps 6 and 7 at least twice, so that you have at least four values of weight (force) and acceleration.

*The *weight* of an object is a *force*; it is the force exerted on the object by the earth's gravity.

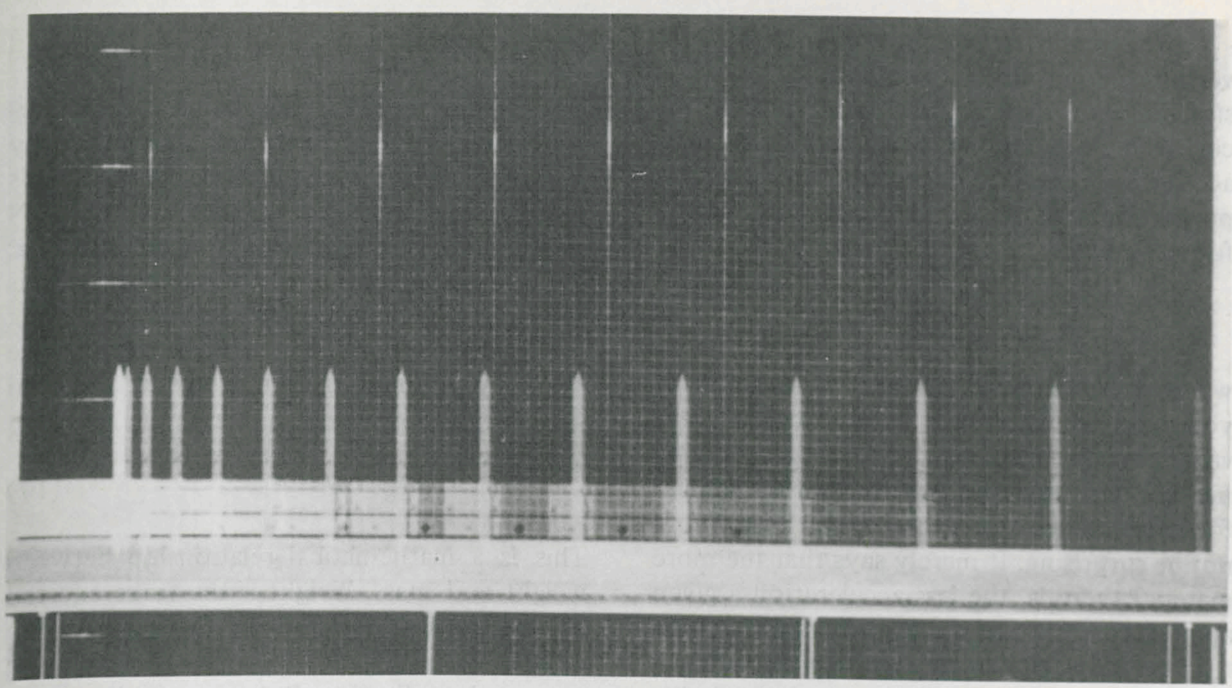


Figure 55.

9. Plot a graph of the hanging weight (applied force in newtons) on the vertical axis and the resulting acceleration (in m/s^2) on the horizontal axis.

10. Compare your graph with those of other students who used different values of total mass. In particular, how do the slopes of the graphs differ?

The graph of weight (applied force F) versus the resulting acceleration (a) is a straight line through the origin. Thus we conclude that, so long as the total mass accelerated remains the same, the force is proportional to the acceleration. In equation form, we write

$$F = ka$$

where k is the slope of the F vs a graph. If you now compare such graphs for different values of the total mass (glider plus hanging masses), you will find that the slope is steeper for larger values of the total mass. This should not be surprising. It merely says that the more material there is, the less acceleration a given applied force produces. Thus it might appear that the slope of the force-acceleration graph is related to the mass being accelerated. We can *define* the mass of an object to be the slope of such a graph. Mathematically, this means replacing k with m, the mass:

$$F = ma \tag{15}$$

A mass of unit size requires a force of one unit to produce a unit of acceleration. Rewriting Equation (15) gives us the mass in terms of the applied force and the resulting acceleration:

$$m = \frac{F}{a} \tag{16}$$

Equation (15) is called *Newton's second law.* It may seem strange to discover a "second law" before learning about a first law, but the numbering only reflects the historical development of the laws. *Newton's first law* says that if no forces act, then there is no acceleration. In the absence of forces, objects continue to do whatever they are doing. This is exactly what Equation (15) implies if $F = 0$.

Equation (15) is valid only if the units used are *consistent.* That is, the mass m must be expressed in the same units as the ratio of force F to acceleration a. In SI units, mass is in kilograms (kg), acceleration is in meters per second per second (m/s^2), and force is in newtons (N). Thus, from Equation (15),

$$1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$$

You are probably accustomed to specifying the amount of matter in an object by stating the *weight* of the object. At the earth's surface a 1-kg mass weighs about 9.8 N (2.2 lb). In empty space, it weighs nothing; but it is still 1 kg of mass. Since weight (W) is force due to gravity, and g is the resulting acceleration due to the force of gravity, we can write Equation (17), which is a special case of Equation (15):

$$W = mg \tag{17}$$

This is a mathematical relationship between weight and mass. Weight is the force exerted by the gravitational pull of the earth on an object. The acceleration of gravity which you measured in Section B is the result of the gravitational force acting on the falling ball. Your weight is the result of the gravitational force acting on your body. Near the earth's surface, an object's weight gives it an acceleration of $g \cong 9.8$ m/s^2.

Several forces may act together in such a way that all the forces add to zero (there is no *net* force). Then the effective force is zero and the acceleration is also zero. This condition is called *equilibrium.* A book on a table is in equilibrium because its weight downward plus the table's push upward give a zero net force. A glider moving along a level air track without friction moves at constant velocity (no net forces), and it is also in equilibrium.

The following examples illustrate the mathematical use of Newton's second law.

Example 12. Find the force which must act on a book with a mass of 1 kg in order to give it an acceleration of 9.8 m/s^2.

Solution. The force necessary to give a 1-kg book an acceleration of 9.8 m/s^2 is

$$F = ma = 1 \text{ kg} \times 9.8 \text{ m/s}^2$$

$$= 9.8 \text{ kg}\cdot\text{m/s}^2 = 9.8 \text{ N}$$

This is the force that gravity exerts on the book; it is equal to the weight of the

book. This example also provides some feel for the size of the newton as a force unit. On the earth's surface, every kilogram of mass is acted upon by a gravitational force of 9.8 N. The next time you are in the lab, pick up what you *used* to call a 1-kg *weight* and say to yourself that it is really a 1-kg mass which has a weight of 9.8 N. Spring scales, which really measure force, should be calibrated in newtons. Figure 56 shows the re-calibration which should be done to scales which read in kilograms. The kilogram is a unit of mass; the newton is the corresponding unit of weight or force in the SI system of units.

THE ENGLISH SYSTEM OF UNITS

It is also useful to consider some of these quantities in comparison to the commonly used English (foot-pound-second) system. We have hopes that the U.S.A. will someday soon "go metric," as has Great Britain. Until then, we are stuck with using the difficult English system for many everyday measurements. We know from measurements which we can make that a one-kilogram mass weights 2.2 lb. This may give you a better feeling for the size of the newton. A force of 9.8 N is equivalent to a force of 2.2 lb.

We have not yet mentioned the unit of mass in the English system. It is called the *slug* and is defined by measuring forces in pounds and accelerations in feet per second per second, as illustrated in Example 13.

Example 13. A mass dropped at the earth's surface accelerates with a constant acceleration of 32 ft/s^2 (9.8 m/s^2). What mass will be given this acceleration by a one-pound force? (This is the same as asking what mass has a *weight* of one pound.)

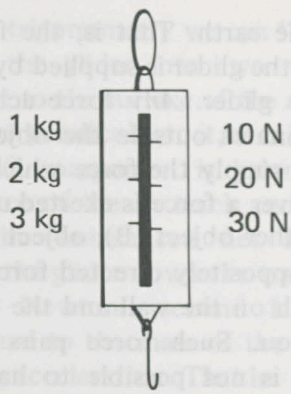


Figure 56.

Solution. We have to remember to stay in a consistent set of units, in this case the foot-pound-second system. A force of 1 lb acts on an unknown mass to give it an acceleration of 32 ft/s^2. What is the mass?

$$F = ma = 1 \text{ lb} = m \times 32 \text{ ft/s}^2$$

or

$$m = \frac{1 \text{ lb}}{32 \text{ ft/s}^2} = \frac{1}{32} \frac{\text{lb} \cdot \text{s}^2}{\text{ft}} = \frac{1}{32} \text{ slug}$$

A unit of mass equal to one pound (second)^2/foot is called the *slug*. A 1-lb weight has a mass of 1/32 slug, or we could also say that a mass of 1 slug weighs 32 lb. Table X illustrates this relationship.

NEWTON'S THIRD LAW

Referring to Experiment C-1, the force on the glider is supplied by the string. The force on the string is provided by the weight of the suspended mass which, in turn, comes from the gravitational force on the hanging

Table X.

	Mass	Acceleration	Force
SI	(kg)	meters/second^2 (m/s^2)	newtons (N)
CGS	(g)	centimeters/second^2 (cm/s^2)	dynes (dyn)
FPS	(lb)	feet/second^2 (ft/s^2)	slugs

mass by the earth. That is, the force which accelerates the glider is supplied by something outside the glider. *Any* force acting on any object originates outside the object; the object cannot supply the force which pushes it.

Whenever a force is exerted on an object (A) by another object (B), object A exerts an equal but oppositely directed force on object B. You push on the wall and the wall pushes back on you. Such force pairs are always present. It is not possible to have a single force acting by itself. Try pulling with a 10-N force on a piece of string not attached to anything else. You can't do it! Usually one of the pair of forces is called an *action* force and the other a *reaction* force. *Newton's third law* asserts that to every action force exerted *by* an object, an equal and oppositely directed reaction force is exerted *on* the object. (Note that the two forces of an action-reaction pair *always* act on two *different* objects.)

We have been very glib in asserting the validity of Newton's laws. The laws have *not* been derived or experimentally determined in a rigorous manner in this module. Their usefulness and reasonableness have been treated only briefly. However, the experiences and experiments of ourselves and others over a period of many years gives us confidence in the correctness of these laws. Here, the essential things to grasp are the concepts of force and mass. Applying Newton's second law to complicated real-life situations is often difficult and sometimes impossible. Some simplified but important applications are illustrated in the following problems.

Problem 23. When a car hits a brick wall, it is rapidly decelerated by the force of the wall. This deceleration is not instantaneous, but takes a small amount of time as the sheet metal and frame crumple.

a. Make the assumption that a car hitting a wall at 100 km/h (about 62 mph or 28 m/s) will travel about 1.22 m (about the length of the hood) before it is brought to rest, and calculate the average acceleration that the car must have undergone.

b. Calculate the average force that the wall exerted on the car, assuming a reasonable mass for the car.

c. Calculate the length of time required for the car to be stopped.

d. In addition to leaping tall buildings at a single bound, Superman was able to bring speeding trains to a dead stop with his bare hands. Even if Superman could exert such a force, and not be harmed, what would happen to the train? Explain your answer.

Problem 24. The mass of a Honda Civic is about 700 kg. If it has run out of gas and the truck which stops and offers a tow has only a 50-kg log chain with which to tow:

a. What force will the *chain* have to exert *on the Honda* to give it an acceleration of 3 m/s^2?

b. What force will the *truck* have to exert *on the end of the chain* to give the chain plus the Honda the acceleration of 3 m/s^2?

c. Explain why the truck has to pull harder than the force felt by the Honda.

Problem 25. As a general rule, it is not too bad an approximation to say that the largest horizontal force which can be applied between the ground and the tires of a car is about equal to the weight of the car. In that case, what is the greatest deceleration (negative acceleration) with which an 18,700-N (about 4200-lb) Cadillac can stop? What is the greatest deceleration with which a 10,700-N (about 2400-lb) Volkswagen can stop?

THE TILTED AIR TRACK (OPTIONAL)

In Experiment B-1 you may have taken data for a glider accelerating down a tilted air track. Analysis of those data would show a

uniform acceleration down the track. Let us consider this motion.

Up to now, we have only considered linear motion problems where not only was the motion in a straight line, but that line was either vertical or horizontal. For the tilted air track the forces which act are a little more complicated. We can analyze the system by breaking the forces up into parts, called components, as was done for velocity in Example 11. In this case, we will look for components which are parallel, or perpendicular, to the line along which the motion occurs—the surface of the air track. The advantage of this procedure is that, because the glider has no motion in a direction perpendicular to the track, we know that the component of the glider's weight in that direction must be just balanced by the upward push of the track on the glider. That is, in the direction perpendicular to the track the forces add to zero, giving no net force. (In the direction perpendicular to the track, the glider is in equilibrium.) Figure 57 illustrates this.

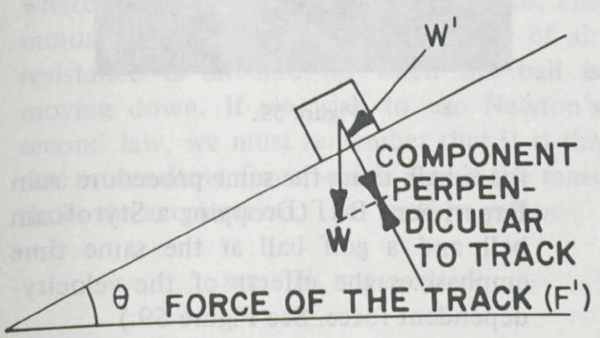


Figure 57. When the glider is on the air track, W', the component of the weight which is perpendicular to the track is always balanced by F', the upward force of the track on the glider.

But there is also a component of the glider weight which is parallel to the track, as shown in Figure 58A. Because the track is "frictionless," no forces parallel to the track are exerted on the glider by the track. Thus the glider experiences a net non-zero force down the track. This force produces an acceleration.

Using trigonometry we can "resolve" the weight of the glider into two components, one parallel to the surface of the air track and the other perpendicular to it, as in Figure 58B. The $mg\cos\theta$ component is balanced by the upward force of the air track and the $mg\sin\theta$ provides the unbalanced net force which accelerates the glider down the track.

From Newton's second law, the net force is equal to the mass of the glider times its resulting acceleration. That is

$$F = mg\sin\theta = ma$$

Dividing by m, we have

$$a = g\sin\theta \tag{18}$$

Since the sine of an angle is a number whose size is between zero and one, the acceleration of the glider is between zero and the acceleration of gravity. The acceleration is zero if $\theta = 0°$ ($\sin 0° = 0$ and the track is level), equal to g if $\theta = 90°$ ($\sin 90° = 1$ and the track is vertical).

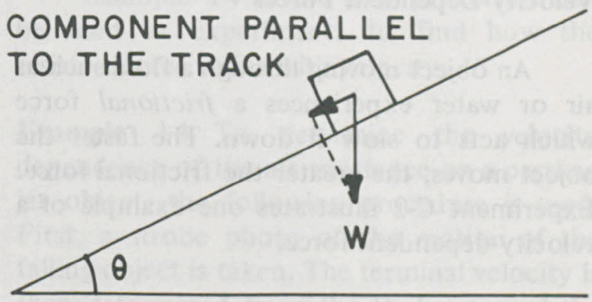


Figure 58A. There is a component of the weight of the glider down the track.

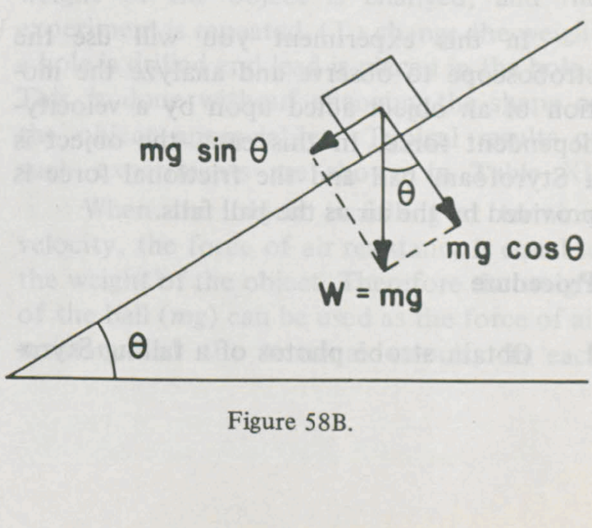


Figure 58B.

NON-UNIFORM ACCELERATION

The rest of this module is intended to show a few applications of Newton's laws to slightly more complicated real-life problems. So far this module has emphasized the motion which results when a constant force is applied to an object. According to Newton's second law, a *constant* force produces a *constant* (or uniform) *acceleration.* We have emphasized uniform acceleration for two reasons.

1. It is a very common form of motion.
2. The mathematics needed to analyze uniform motion is not so difficult as it is for more complicated motion.

There are many interesting cases where the force is not constant and the resulting acceleration is not uniform. We will examine the general features of two particular kinds of non-uniform acceleration and will describe how to proceed with the mathematics required to analyze the motion.

Velocity-Dependent Forces

An object moving through a fluid such as air or water experiences a *frictional* force which acts to slow it down. The faster the object moves, the greater the frictional force. Experiment C-2 illustrates one example of a velocity-dependent force.

EXPERIMENT C-2. Velocity-Dependent Forces

In this experiment you will use the stroboscope to observe and analyze the motion of an object acted upon by a velocity-dependent force. In this case, the object is a Styrofoam ball and the frictional force is provided by the air as the ball falls.

Procedure

1. Obtain strobe photos of a falling Styrofoam ball, using the same procedure as in Experiment B-1. (Dropping a Styrofoam ball and a golf ball at the same time emphasizes the effects of the velocity-dependent force. See Figure 59.)

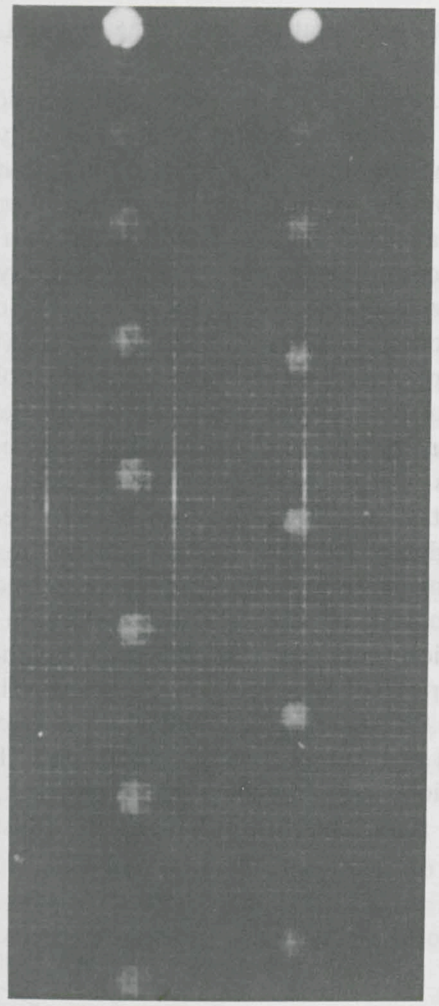


Figure 59.

2. Look at the spacing of the images. Is it uniform? If not, repeat the experiment, starting from a greater height. To get uniform spacing you may have to throw the balls downward instead of just dropping them.
3. As in Experiment B-1, make a graph of velocity versus time. Does the velocity become constant? If so, record this value as the *terminal velocity.* Is the accelera-

tion constant for any part of the graph?

4. Record in a table the values of diameter, mass, and terminal velocity for each ball you photograph. You may want to trade information with other students to make a more complete table.

5. Study the data in the table to see if you can detect any pattern. You may wish to plot terminal velocity versus mass or terminal velocity versus diameter to help your analysis.

Analysis of Experiment C-2.

When a ball falls through the air, as in Experiment C-2, two forces act on it: the constant force of gravity in the downward direction and the velocity-dependent upward force of air resistance. The total force F_t is

$$F_t = mg - F_v \quad (19)$$

where F_v is the velocity-dependent force. The minus sign appears because the force of air resistance is directed up when the ball is moving down. If we wish to use Newton's second law, we must remember that it is the net or *resultant* force which equals the mass times the acceleration. That is, $F_t = ma$, or

$$mg - F_v = ma$$

Three possibilities arise:

1. $F_v = mg$. If the force of air resistance equals the weight of the ball, the resultant force is zero and the ball falls with constant velocity. This constant velocity is called the *terminal velocity*.

2. F_v less than mg. In this case the force of air resistance is less than the weight. The net force is downward, so the acceleration is downward. Therefore, the velocity increases, but at a lesser rate than it would if no air were present. If, as it speeds up, the air resistance increases, it will sooner or later get big enough so that $F_v = mg$. Then it will travel at a constant terminal velocity.

3. F_v greater than mg. If the force of air resistance is greater than the weight, the net force is upward. This means that the acceleration is also upward. Then the downward velocity of the ball decreases until it reaches terminal velocity. Throwing the ball downward with a velocity greater than the terminal velocity produces this situation. The frictional force of air resistance is greater than the weight of the ball at first. Thus, the ball slows down until it reaches terminal velocity.

The important point is that, no matter what the initial velocity, the ball eventually reaches terminal velocity. Judging from your strobe photos, which of the three situations have you produced?

Example 14 shows how this effect can be used in experiments to find how the resistive force varies with velocity.

Example 14. To determine the velocity dependence of the air resistance on a particular object, the following procedure is used. First, a strobe photo of the motion of the falling object is taken. The terminal velocity is then determined from the flash rate and the image spacing, as in Experiment C-2. The weight of the object is changed, and the experiment is repeated. (To change the weight a hole is drilled and lead is placed in the hole.) This is done without changing the shape of the object appreciably. Typical results of such experiments are shown in Table XI.

When the object is falling at terminal velocity, the force of air resistance is equal to the weight of the object. Therefore the weight of the ball (mg) can be used as the force of air resistance at the terminal velocity in each

Table XI.

Total weight (N)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Terminal velocity (cm/s)	41	73	100	124	145	165	183	200	216	232

case. This system was used to produce the graph of Figure 60.

The graph was produced using information collected for objects moving at terminal velocity. However, the force of air friction is also correct for any velocity and any mass, provided the size and shape of the object remains the same.

This allows us to determine the acceleration for any values of weight and velocity. For example, suppose $W = 0.5$ N and $v = 80$ cm/s. Reading from the graph, if the object is moving at 80 cm/s, the force of air resistance is 0.22 N. So the net downward force on the object is the weight minus the frictional force, or

$$F_t = mg - F_v$$

$$= 0.5 \text{ N} - 0.22 \text{ N} = 0.28 \text{ N}$$

Apply Newton's second law to calculate the acceleration:

$$a = \frac{F_t}{m}$$

and

$$m = \frac{W}{g}$$

Thus

$$a = \frac{F_t}{W/g} = \frac{F_t}{W} g = \frac{0.28 \text{ N}}{0.5 \text{ N}} \times 9.8 \text{ m/s}^2$$

$$= 5.5 \text{ m/s}^2$$

This example illustrates one way to collect information about a velocity-dependent force. It also shows how to use that information to calculate the acceleration. You might wonder if we also could determine how far an object will fall in a given time.

In this case we cannot use the equations of motion for constant acceleration, since the acceleration is *not* constant. However, we can do an *approximate* calculation based on these equations. The approximation assumes that the acceleration is nearly constant for very small time intervals. Figure 61 is a *flow chart* of the procedure. This kind of *iterative* (repeating) procedure is typically used in computer solutions to physics problems. Figure 62 shows computer calculated values of position, velocity, and acceleration over a one-second period for a ball initially falling at a speed of 80 cm/s.

The procedure outlined in Figure 61 is also applicable in other areas of physics. Such iterative methods are very powerful tools for the analysis of many different physics problems.

Problem 26. Discuss why the graphs in Figure 62 behave as they do as the ball approaches terminal velocity.

Problem 27. A few very lucky people have fallen from airplanes without parachutes and have lived.

a. Discuss what part air resistance (and soft ground!) may have played in their survival.

b. Discuss in terms of the physics of this section why a parachute is effective.

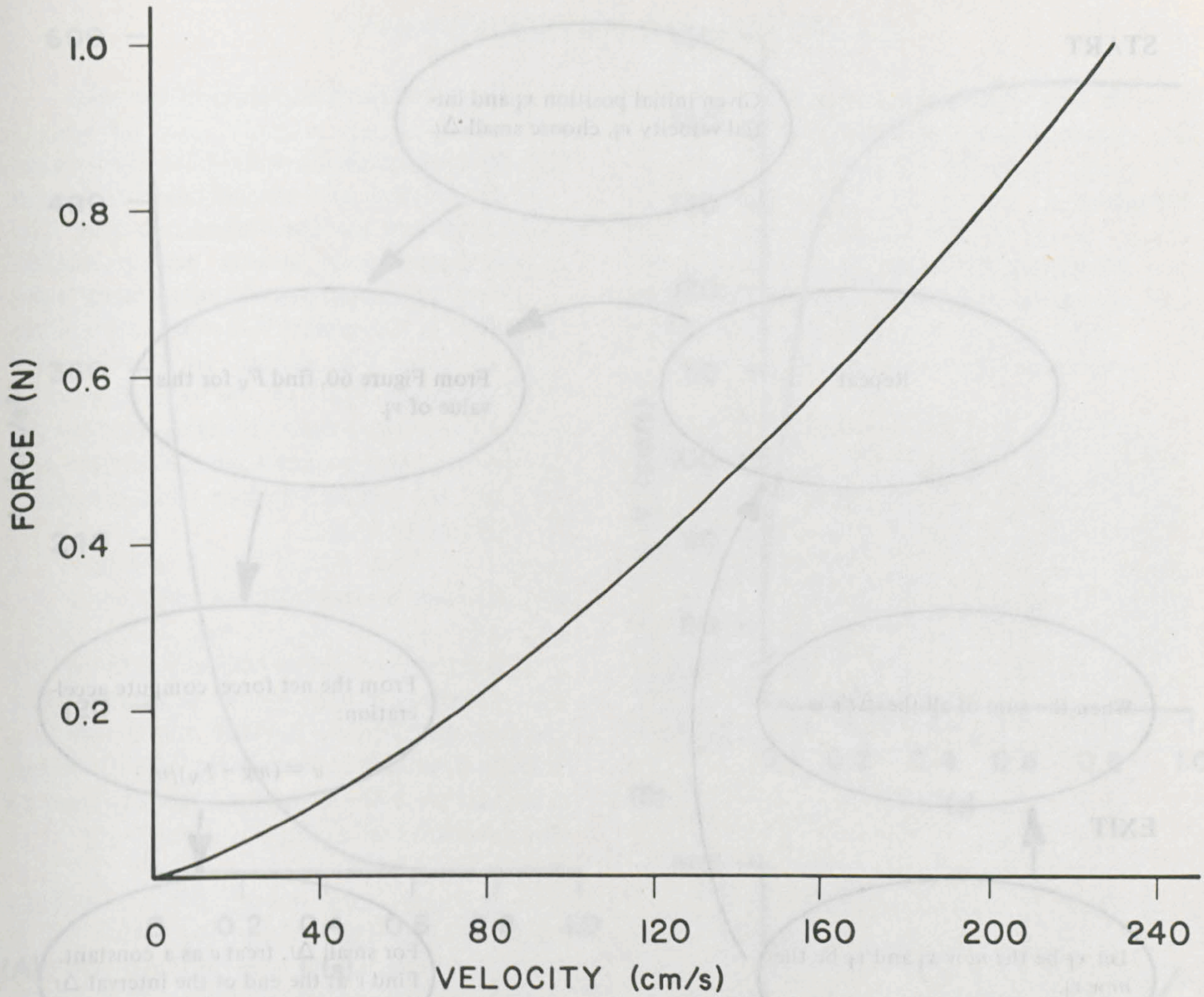


Figure 60.

Problem 28. Figure 59 shows two white balls falling past the string grid. The strings are 1 cm apart. The strobe flash rate is 1200 fpm.

a. Plot position-time curves on the graph provided.

b. Sketch the corresponding velocity-time curves roughly. (Don't bother to calculate points.)

c. Are the accelerations of the balls the same? Are they both *uniformly accelerated*?

d. What does this say about the net force on each of the balls?

e. Analyze the forces on each ball to discuss the net force on each of them. (What is the cause of each of the forces? Are all of the forces constant? What do they depend on?) How do you suppose the balls differ? Which ball is heavier? Can you relate this problem to the parachute in Problem 27?

Position-Dependent Forces

Another important group of forces includes those forces which depend on the position of the object experiencing the force. Many everyday examples of such *position-dependent* forces exist. Both Experiments C-3 and C-4 involve such forces.

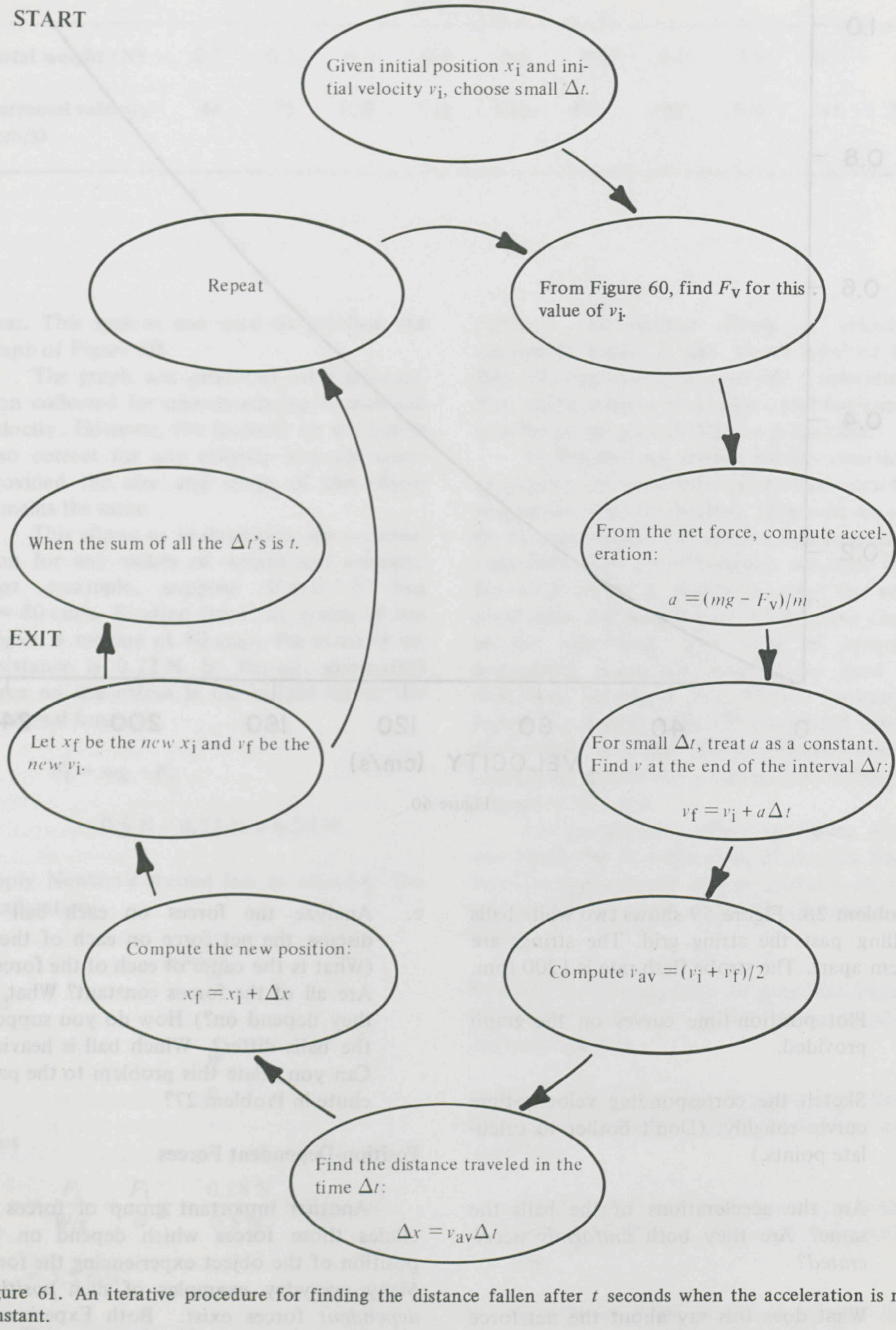
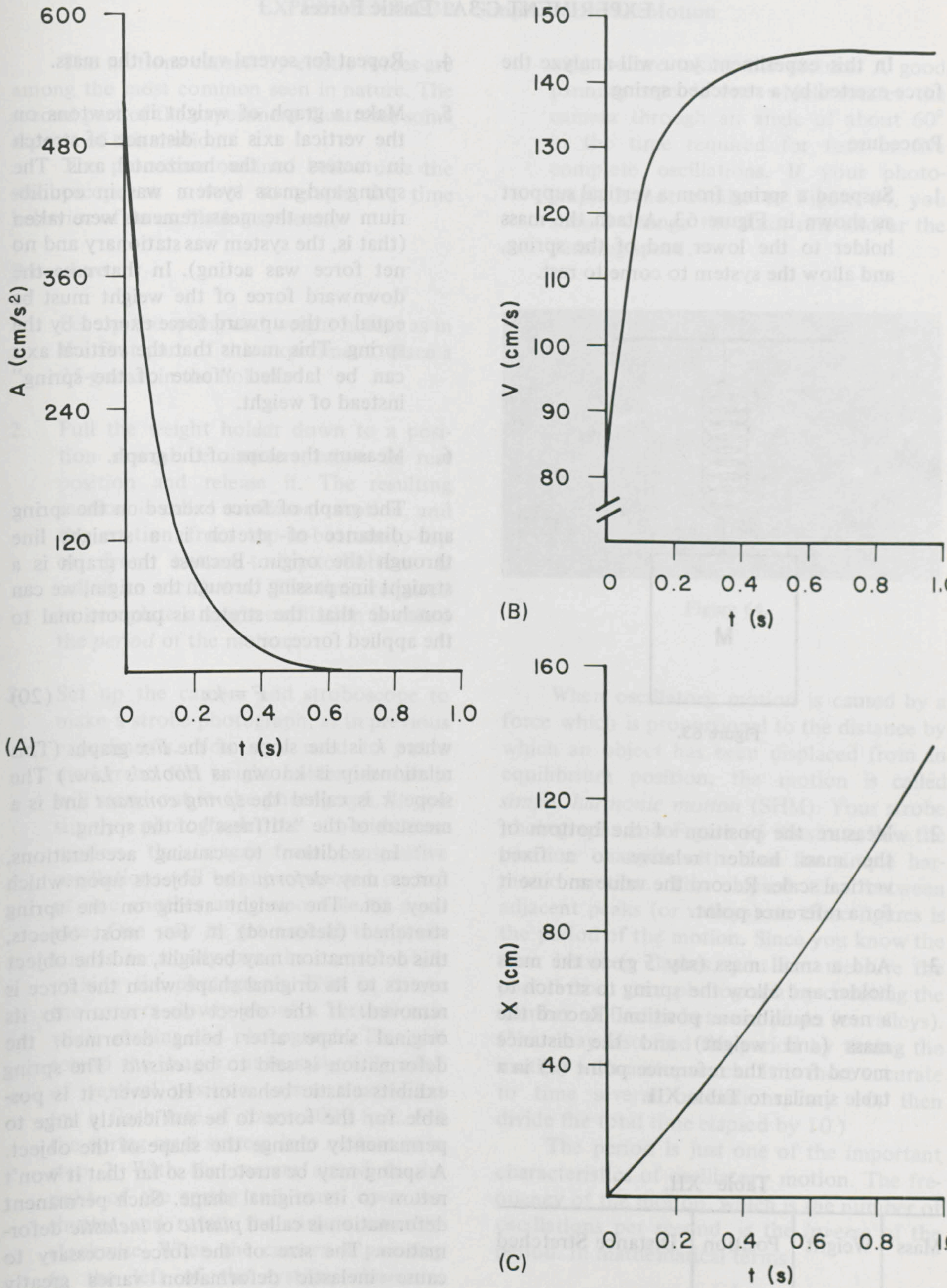


Figure 61. An iterative procedure for finding the distance fallen after t seconds when the acceleration is not constant.



600
480
360
240
120
A (cm/s²)
0
0.2
0.4
0.6
0.8
1.0
t (s)
(A)
150
140
130
120
110
100
90
80
V (cm/s)
0
0.2
0.4
0.6
0.8
1.0
t (s)
(B)
160
120
80
40
X (cm)
0
0.2
0.4
0.6
0.8
1.0
t (s)
(C)

Figure 62.

EXPERIMENT C-3A. Elastic Forces

In this experiment you will analyze the force exerted by a stretched spring.

Procedure

1. Suspend a spring from a vertical support as shown in Figure 63. Attach the mass holder to the lower end of the spring, and allow the system to come to rest.

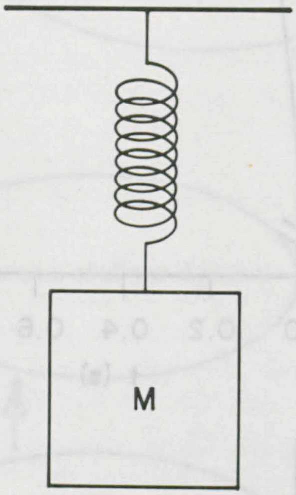


Figure 63.

2. Measure the position of the bottom of the mass holder relative to a fixed vertical scale. Record the value and use it for a reference point.

3. Add a small mass (say 5 g) to the mass holder and allow the spring to stretch to a new equilibrium position. Record the mass (and weight) and the distance moved from the reference point (x) in a table similar to Table XII.

Table XII.

Mass	Weight	Position	Distance Stretched
kg	N	m	m

4. Repeat for several values of the mass.

5. Make a graph of weight in newtons on the vertical axis and distance of stretch in meters on the horizontal axis. The spring-and-mass system was in equilibrium when the measurements were taken (that is, the system was stationary and no net force was acting). In that case the downward force of the weight must be equal to the upward force exerted by the spring. This means that the vertical axis can be labelled "force of the spring" instead of weight.

6. Measure the slope of the graph.

The graph of force exerted on the spring and distance of stretch is a straight line through the origin. Because the graph is a straight line passing through the origin, we can conclude that the stretch is proportional to the applied force, or

$$F = kx \qquad (20)$$

where k is the slope of the F-x graph. (This relationship is known as *Hooke's Law.*) The slope k is called the *spring constant* and is a measure of the "stiffness" of the spring.

In addition to causing accelerations, forces may *deform* the objects upon which they act. The weight acting on the spring stretched (deformed) it. For most objects, this deformation may be slight, and the object reverts to its original shape when the force is removed. If the object does return to its original shape after being deformed, the deformation is said to be *elastic.* The spring exhibits elastic behavior. However, it is possible for the force to be sufficiently large to permanently change the shape of the object. A spring may be stretched so far that it won't return to its original shape. Such permanent deformation is called *plastic* or *inelastic* deformation. The size of the force necessary to cause inelastic deformation varies greatly among different materials.

EXPERIMENT C-3B. Simple Harmonic Motion

The motions caused by elastic forces are among the most common seen in nature. The second part of the experiment illustrates some aspects of this motion.

The procedure outlined below uses the stroboscope as a tool to graph the time behavior of an oscillating system.

Procedure

1. Set up the spring and weight holder as in the first part of this experiment. Place a 25-g mass in the holder.

2. Pull the weight holder down to a position several centimeters below its rest position and release it. The resulting motion is called *oscillatory* motion, and the motion from top-to-bottom-to-top or from bottom-to-top-to-bottom is called an *oscillation.* The time required to complete a single oscillation is called the *period* of the motion.

3. Set up the camera and stroboscope to make a strobe photograph, as in previous experiments. You may need to put a marker on the weight holder so that it will stand out in the photograph. If you simply photograph the up-and-down motion, the images from consecutive oscillations will be superimposed on top of one another and impossible to separate. One way to get around this problem is to slowly *pan* the camera while making the photograph. That is, rotate the camera slowly about a vertical axis while making the photograph. This will spread the images out to make a "graph" of vertical position versus time. First try a flash rate of about 1000 fpm. Set the spring-mass system into motion as in step 2. With the camera aimed to the right of the spring and mass, open the shutter and rotate the camera counterclockwise. When the camera is pointing to the left of the system, close the shutter. You may wish to practice panning before taking the picture. A good panning rate is one which rotates the camera through an angle of about 60° in the time required for four or five complete oscillations. If your photograph is not similar to Figure 64, you should change the flash rate and/or the panning speed.

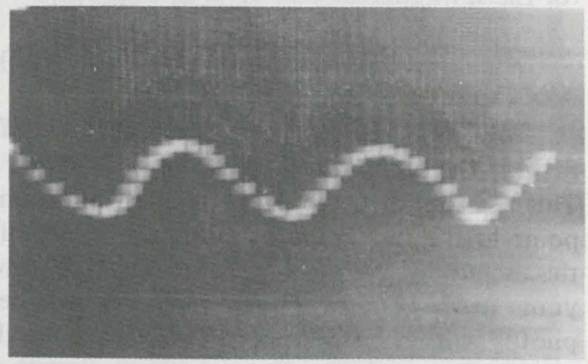


Figure 64.

When oscillatory motion is caused by a force which is proportional to the distance by which an object has been displaced from an equilibrium position, the motion is called *simple harmonic motion* (SHM). Your strobe photographs and Figure 65 illustrate how the position changes with time for simple harmonic motion. The elapsed time between adjacent peaks (or valleys) on these figures is the period of the motion. Since you know the time between flashes, you can measure the period from your photograph by counting the number of flashes between peaks (or valleys). (You may also find the period by timing the motion with a stopwatch. It is more accurate to time several oscillations, say 10, then divide the total time elapsed by 10.)

The period is just one of the important characteristics of oscillatory motion. The frequency of the motion, which is the number of oscillations per second, is the *inverse* of the period. In mathematical terms

$$T = \frac{1}{f} \qquad (21)$$

where T is the period and f is the frequency. The period depends primarily on two quantities. One of these is the amount of mass to be moved. The greater the mass, the more slowly it will oscillate. (Try this by hanging different masses from a spring and timing the oscillations.) The second is the spring constant. The greater the spring constant, the faster the system will oscillate. (Try this by using different springs.) Measurements of the dependence of T on m and k establish that, for the spring-mass system,

$$T = 2\pi\sqrt{m/k} \qquad (22)$$

Another important characteristic of SHM is the *amplitude* (A) of the oscillations. This is the distance from the equilibrium point to the maximum or minimum height of the oscillating mass. It is easily measured from your strobe photo. If you compare strobe photos with other persons who have used springs similar to yours, you can see that the period does *not* depend on the amplitude. Large oscillations take the same time as small ones.

Figure 65 relates period and amplitude to the shape of the curve traced out in the strobe photograph in Figure 64. A graph the shape of Figure 65 is called a *sine* curve. The formula relating vertical position x to time t can be written as $x = A\sin(360° \, t/T)$. Such a relation is called a *sinusoidal function.* The position-time graph of a motion is sinusoidal if that motion is simple harmonic motion.

Figure 66 is a strobe photograph of another common type of SHM. It illustrates one-half of the oscillation of a *pendulum.* You might find it interesting to compare this to your strobe photo of the oscillating mass. For each system at what points in the cycle is the motion slowest? Fastest? Could the motion of the pendulum be graphed in the same way as the motion of the spring?

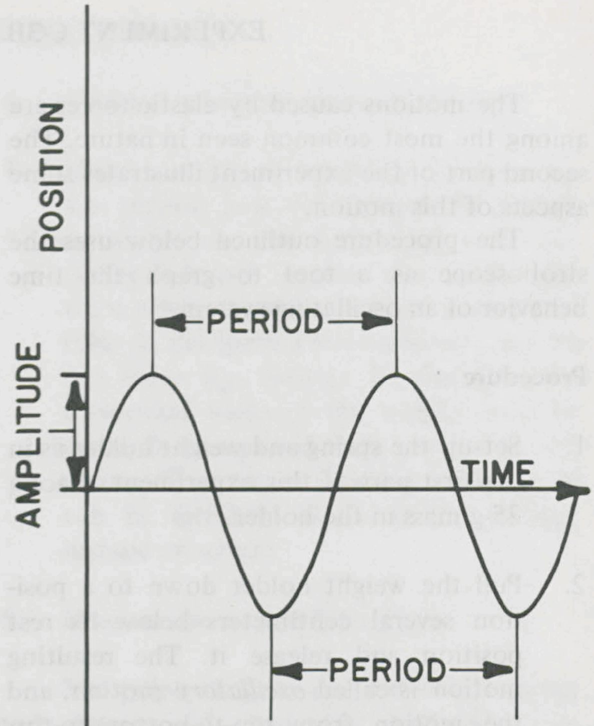


Figure 65.

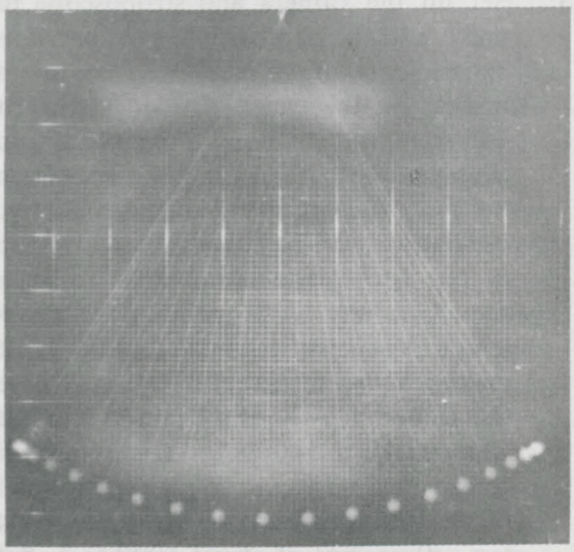


Figure 66.

EXPERIMENT C-4 (OPTIONAL). Another Position-Dependent Force

In this experiment you will observe the motion of an object acted upon by one type of position-dependent force.

Procedure

1. Level the air track.

2. With the glider at the pulley end of the track, attach a light string to the glider. Make the string just long enough to reach the floor. Fasten a length of flexible chain to the string. The chain should be at least long enough to reach the floor when the glider is at the opposite end of the track. The setup is shown in Figure 67.

3. Use a stopwatch to determine the time required for the glider to travel the full length of the track. (If you have more than one glider, use the longest one first.) Record the mass of the glider and the travel time.

4. Repeat, starting the glider from the middle of the track. Before doing the experiment, guess what the time will be.

5. Repeat steps 3 and 4 with a larger glider, or add mass to the first glider. Start the front of the glider at the same positions as in the previous runs. Record the mass and the travel time. Before doing the experiment you should guess what the time will be.

6. Enter your data in a table showing glider mass, distance traveled, and time. If you can, get additional data from other students.

7. Take a strobe photo for one of the cases. Select a flash rate to give you about 12 images. The proper rate can be calculated from the known time of travel.

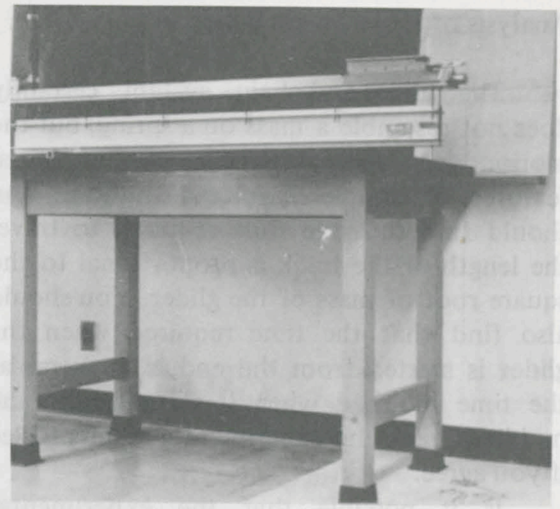


Figure 67A. With the glider at one end of the air track, all of the chain is on the floor.

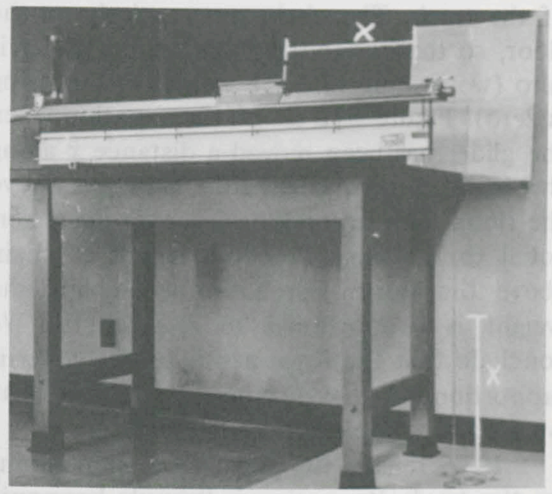


Figure 67B. The length of chain off the floor is equal to the distance of the glider from the end of the track.

8. Plot position-time and velocity-time graphs from the information on the strobe photo.

9. Describe your observations and list any tentative conclusions you have reached.

Analysis of Experiment C-4

The glider-and-chain system certainly does not resemble a mass on a spring, but the motion does agree with two of the general features of simple harmonic motion. You should find that the time required to travel the length of the track is proportional to the square root of mass of the glider. You should also find that the time required when the glider is started from the end is the same as the time required when it started from the middle. Check the experimental results to see if you agree.

Is it possible that the experimental arrangement in Experiment C-4 provides a force of the type needed to produce simple harmonic motion? Figure 67A shows the situation when the glider is at the pulley end of the track. The chain rests entirely on the floor, so the horizontal force on the glider is zero (we assume that the weight of the string is zero). Figure 67B shows the situation when the glider has been moved a distance x along the track. Now a length of chain x is above the floor. This produces a force on the glider, equal to the weight of the length of chain above the floor. For a uniform chain, the weight is proportional to the length. We conclude that the force acting on the glider is proportional to the distance x, which is what we need for simple harmonic motion.

If we release the glider from some point on the track, its motion is "simple harmonic motion" until it reaches the end. Then things change because a new force acts on the glider–the force exerted by the bumper at the end of the track. However, until that is reached, the glider moves through part of a cycle of simple harmonic motion. What part?

When we release the glider from rest, it moves from its extreme position at $x = -A$ to the $x = 0$ position (where $F = 0$). This is the first quarter of a cycle of simple harmonic motion. Therefore, the time you measured in Experiment C-4 is one-quarter of a period.

In particular, the period is

$$T = 2\pi\sqrt{\frac{ML}{mg}}$$

where M is the mass of the glider and mg is the weight of a length of chain, L. (Don't forget that you measured one-quarter of a period.)

The position of the glider at any time is given by

$$x = A\sin(360^\circ\ t/T)$$

where A is the maximum displacement from the position where $x = 0$. (That is, A is the distance from the pulley end of the track to the point where you released the glider.) In applying this, x must be measured from the point where $F = 0$ (that is, the end of the track when all the chain is on the floor). The time t is measured from the time of release.

We will not analyze this motion further, but we state without proof that it is identical to simple harmonic motion in every respect except that it does not repeat itself.

SUMMARY

The acceleration of an object is proportional to the *net* force acting on it and inversely proportional to its mass. This is *Newton's second law* and is expressed mathematically as

$$F = ma$$

Fluids exert a *resistive force* on objects which move through them. The frictional force depends on the velocity of the object and its shape and size. When a falling object reaches such a speed that the resistive force equals its weight, the velocity remains constant. This is called the *terminal velocity*.

When the force on an object varies, the acceleration also varies and the motion is complicated. However, if the forces do not change much during small intervals, the entire motion can be studied. This is done by breaking the time into small intervals during which the force is *approximately* constant. Then *iterative* methods can be applied using the values of position and velocity at the end of one interval as the values for the beginning of the next interval.

The force exerted on an object may depend upon its position. Common examples are the motion of a mass on a spring and the motion of a pendulum. If an object oscillates about a rest position because of a force which is proportional to the distance of the object from the rest position, the object is undergoing *simple harmonic motion.* Simple harmonic motion has the following features:

a. The period is inversely proportional to the square root of the "spring constant."

b. The displacement is given by:

$$x = A\sin(360° \; t/T)$$

where T is the *period.*

c. The period is the same for all amplitudes.

For the case of the mass on a spring, the restoring force is given by

$$F = kx$$

where k is the spring constant and x is the displacement. The period is given by

$$T = 2\pi\sqrt{m/k}$$

0-07-001722-0